For two study areas in Minnesota, USA, one heavily forested and one sparsely forested, maps of predicted proportion forest area were created using Landsat Thematic Mapper imagery, forest inventory plot data, and two prediction techniques, logistic regression and a k-Nearest Neighbours technique. The maps were used to increase the precision of forest area estimates by using them as the basis for stratified estimation. Estimates of mean proportion forest area were similar for all estimation methods, but the variances of stratified estimates were smaller than variances under an assumption of simple random sampling by factors as great as 6.

Keywords and phrases: k-Nearest Neighbours, logistic model, stratification

1.0 INTRODUCTION

The five regional Forest Inventory and Analysis (FIA) programs of the Forest Service, U.S. Department of Agriculture, are required to report estimates of forest area for their respective regions every five years. Traditionally, the estimates have been obtained as products of total area inventoried and the mean over a systematic array of field plots of the proportion of each plot in FIA-defined forest land. The FIA definition of forest land includes commercial timberland, some pastured land with trees, forest plantations, unproductive forested land and reserved, non-commercial forested land. In addition, forest land must satisfy minimum stocking levels, a 0.405-hectare (1-acre) minimum area, and a minimum continuous canopy, bole-to-bole width of 36.58 m (120 ft) and, therefore, excludes lands such as wooded strips, idle farmland with trees, and narrow windbreaks. Due to budgetary constraints and natural variability among plots, sample sizes sufficient to satisfy national FIA precision standards are seldom achieved for most inventory variables. Thus, ancillary data in the form of aerial photography (Hansen 1990) or satellite imagery (McRoberts et al. 2002) have been used to create strata to increase the precision of estimates with stratified estimation.

When satellite imagery is used as the basis for stratifications, image pixels are grouped into similarity classes on the basis of predictions of land cover attributes assigned to them, and the classes are then used as strata in the stratified estimation. If the stratification is accomplished prior to sampling and the within-stratum variances of the inventory variables are well-estimated, then maximum precision may be achieved by designing within-stratum sampling intensities to be proportional to within-stratum variances. However, even when the within-stratum sampling intensities are independent of the stratification, stratified estimation may still yield increases in precision.

A regional FIA program on a 5-year plot measurement cycle that uses TM imagery to enhance estimation will need to process approximately 125 images over the cycle and will require sufficient training data in close temporal proximity to the imagery dates to guide the classifications. These are not insignificant tasks, and investigation of efficient means of obtaining training data and processing images are worthwhile FIA endeavours. Thus, the objectives of the study are twofold: (1) to evaluate approaches to constructing maps of proportion forest area using TM imagery and inventory plot data; and (2) to assess map accuracy with respect to the precision of stratified estimates of forest area.
2.0 DATA

The study was conducted in two areas in Minnesota, USA, designated St. Louis and St. Cloud (Figure 1). The St. Louis study area encompasses most of St. Louis County; includes approximately 2.1 million hectares of which approximately 75 percent is forest land; and is dominated by Aspen-Birch and Spruce-Fir associations. The St. Cloud study area contains the St. Cloud urban area; includes approximately 3.3 million hectares of which slightly more than 20 percent is forest land; and is characterized by prairie agriculture and a diverse mixture of forest lands including both coniferous and deciduous species.

2.1 Satellite imagery

The St. Louis study area is covered by the Landsat TM Path 27, Row 27 scene and includes all of St. Louis County except the northern portion. For this scene, Landsat-7 ETM+ images were obtained for two seasons: autumn (5 November 1999) and summer (31 May 2000). The St. Cloud study area is covered by the Landsat TM Path 28, Row 28 scene. For this scene, Landsat-7 ETM+ images were obtained for three seasons: summer (23 July 1999), autumn (27 October 1999), and spring (3 March 2000). The following attributes pertain to all five images: (1) 30m x 30m pixels from bands 1-5 and band 7; (2) absolute radiance units scaled to 8 bits; (3) processing to level 1G (processing level 08; radiometrically and geometrically corrected using satellite model and platform ephemeris information); and (4) geo-referencing to Albers Equal Area projection, NAD83. In addition, for the St. Louis study area, the November image was rectified using 40 ground control points with resulting root mean square error of 12.1m. The May image was registered to the November image using 26 ground control points and resampled using first-order polynomial and nearest neighbour techniques with resulting root mean square error of 31.9m. For the St. Cloud study area, all three images were rectified using ground control points and digital elevation model terrain correction (processing level 10) and resampled using cubic convolution with resulting root mean square error less than 8.5m. Finally, bands are distinguished using an alphanumeric character representing the first letter of the month of the image and a numeric character designating the band. The context of band references indicates whether they refer to St. Louis or St. Cloud images.
2.2 Inventory plot data

Under the FIA program’s annual inventory system (McRoberts 1999), field plots are established in permanent locations using a systematic sampling design. In each state, a fixed proportion of plots are measured annually; plots measured in a single federal fiscal year (e.g., FY-1999: 1 October 1998 to 30 September 1999) comprise a single panel of plots with panels selected for annual measurement on a rotating basis. In aggregate, over a complete measurement cycle, a plot represents 2,403 hectares. In general, locations of forested or previously forested plots are determined using global position system receivers, while locations of non-forested plots are determined using digitization methods.

Each field plot consists of four 7.31-m radius circular subplots. The subplots are configured as a central subplot and three peripheral subplots with centres located at 36.58 m and azimuths of 0°, 120°, and 240° from the centre of the central subplot. Among the observations field crews obtain are the proportions of subplot areas that satisfy specific ground land use conditions. Subplot-level estimates of proportion forest area are obtained by aggregating these ground land use conditions consistent with the FIA definition of forest land, and plot-level estimates are obtained as means over the four subplots.

For both study areas, measurements for the FY-1999 panel of inventory plots were available. For the St. Louis study area, measurements for 133 plots or 532 subplots were used of which 387 subplots were completely forested, seven subplots were partially forested, and 138 subplots were non-forested. For the St. Cloud study area, measurements for 268 plots or 1072 subplots were used of which 226 subplots were completely forested, 13 subplots were partially forested, and 833 subplots were non-forested.

3.0 METHODS

With stratified estimation, two primary tasks must be accomplished: first, the relative proportion of the population represented by each stratum must be determined, and second, each observational unit must be assigned to a stratum. When using maps constructed from satellite imagery, strata weights are simply the proportions of pixels assigned to strata, and the assignment of observational units to strata is based on the strata assignments of their associated pixels. Thus, stratified estimates are adversely affected by two components of map inaccuracy: inaccuracy in the distribution of pixel data representations with respect to the thematic variable adversely affects strata weights, and inaccuracy in data representations for the particular pixels spatially coincident with observational units adversely affects assignment of those units to strata. Fortunately, map inaccuracy adversely affects only the variances of stratified estimates; it does not contribute to estimation bias.

3.1 Model-based prediction

Because proportion forest area is always in the closed interval [0,1], it is appropriate to select a model with mathematical properties that restricts predictions to the same interval. The logistic model is often used with such data and was selected for this study to describe the relationship between observed proportion forest area for FIA subplots and spectral values of corresponding pixels:

\[
E(Y) = \frac{1}{1 + \exp(\beta_0 + \beta_1X_1 + \ldots + \beta_pX_p)}
\]

where \(E(.)\) denotes statistical expectation, \(Y\) is proportion forest area, \(X_i\) is the spectral value of the \(j^{th}\) TM band, the \(\beta\)s are parameters to be estimated, and \(P\) is the number of spectral bands included in the model.

A three-step process was used to select spectral bands for inclusion in models. First, the terms of (1) were rearranged and transformed to produce the model,

\[
E[\ln(\frac{1}{Y} - 1)] = \beta_0 + \beta_1X_1 + \ldots + \beta_pX_p
\]

where \(\ln(.)\) is the natural logarithm function, \(Y=0.001\) replaces \(Y=0.0\), and \(Y=0.999\) replaces \(Y=1.0\). Second, for all values of \(P\) (\(P=1,2,...,12\) for the St. Louis study area; \(P=1,2,...,18\) for the St. Cloud study area), simple linear regression analyses were used to fit (2) to the transformed observations, \(\ln(Y^{-1} - 1)\), for all \(P\)-band combinations.
For each P-band combination, residual root mean square, $\text{RMS}_r$, was calculated, and the five band combinations with the smallest $\text{RMS}_r$ values were selected, regardless of the corresponding values of P. Third, for the five selected combinations for each study area, (1) was fit to the proportion forest area observations using weighted nonlinear regression where the weights reflected the correlations among observations of subplots within the same plot, and $\text{RMS}_r$ was calculated. For each study area, models using the five selected band combinations and the corresponding parameter estimates obtained from the nonlinear analyses were used to create maps by predicting proportion forest area for all pixels for the two study areas.

3.2 k-Nearest Neighbours prediction

The k-Nearest Neighbours (k-NN) technique is a non-parametric approach to predicting values of point variables on the basis of similarity in a covariate space between the point and other points with observed values of the variables. For this application, consider a TM image pixel to be a point, let $Y_i$ denote proportion forest area for the $i^{th}$ pixel, and let $X_i$ denote its vector of TM spectral values. For a finite number, N, of image pixels of which n correspond to FIA subplots, the data points $(Y_i, X_i)$ may be re-ordered without loss of generality so that $(Y_i, X_i)_{i=1,...,n}$ denote the points corresponding to pixels associated with FIA subplots and $(Y_i, X_i)_{i=n+1,...,N}$ denote the points for the remaining pixels. With the k-NN technique, a prediction for any $Y_{j, j=1,...,N}$ is obtained in two steps:

1. for each $Y_j$ re-order $Y_{ik}, i=1,...,n$ with respect to increasing distance, $d_{ji}$, between $X_j$ and each $X_i$ excluding $Y_i$ from the ordering if $1 \leq i \leq n$, and denote the resulting ordering $(Y_{jk})$;

2. for each $Y_j$, 

$$
\hat{Y}_j = \frac{1}{k} \left( \sum_{i=1}^{k} W_{ji} \right) \left( \sum_{i=1}^{k} W_{ji} Y_{ji} \right)
$$

where k is a predetermined constant, $1 \leq k < n$, and $\{w_{ji}\}$ are point weights to be selected.

The quality of predictions may be assessed using $Y_{ik}, i=1,...,n$, an appropriate objective criterion, and the leaving one-out-method. With the leaving-one-out method, a k-NN prediction is sequentially obtained for each $Y_i$, but with the provision that $Y_i$ itself cannot be included in the mean forming its own k-NN prediction. In addition, to avoid issues related to the expected high correlation expected among attributes for subplots of the same plot, for this study the prediction for a subplot was constrained against including an observation for any of the other subplots of the same plot. By comparing the observations, $Y_{ik}, i=1,...,n$, and the corresponding predictions with respect to the objective criterion, the quality of predictions may be evaluated.

Before implementation, the k-NN technique must be calibrated. First, the particular spectral bands used to calculate the distances, $d_{ji}$, between $X_j$ and each $X_i$, must be selected. Without loss of generality, the bands may be re-ordered so that $X_{ip}, p=1,...,P$ designates the P selected bands. Second, a distance metric, d, must be selected; among the alternatives are weighted Euclidean distance,

$$
d_{ji} = \left( \sum_{p=1}^{P} v_p (X_{jp} - X_{ip})^2 \right)^{1/2}
$$

where $\{v_p\}$ are variable weights, and Mahalanobis distance,

$$
d_{ji} = (X'_j - X'_i)' V^{-1} (X'_j - X'_i)
$$

where only the selected P components of $X$ are used and $V$ is the covariance matrix for the P selected components of $X$. If weighted Euclidean distance is selected, then the variable weights $\{v_p\}$ for (4) must also be selected. Third, the value of k, the number of nearest neighbours to be included in the calculation of predictions (3), must be selected. Finally, the point weights, $\{w_{ji}\}$, for (3) must be selected; common alternatives include constant weighting for which $w_{ji}=1$, inverse distance weighting for which $w_{ji}=d_{ji}^{-1}$, and inverse distance squared weighting for which $w_{ji}=d_{ji}^{-2}$. 
For this application, unweighted Euclidean distance, constant variable and point weighting, and RMS as the objective criterion were selected. For each study area, \( k_{\text{opt}} \), the value of \( k \) that minimized \( \text{RMS}_e \), was determined for each combination of spectral bands by comparing values of \( \text{RMS}_e \). The five spectral band combinations with smallest \( \text{RMS}_e \), without regard to the number of bands, were selected and were used to create maps by predicting forest area proportion for all pixels for the two study areas.

### 3.3 Stratified estimation

The FIA program avoids the mathematical complexity associated with the spatial correlation among the four subplot observations by assigning plots rather than subplots to strata. However, because an FIA plot is associated with multiple TM pixels, the task is not routine. Three approaches to assigning plots to strata are considered. The first approach assigns strata to plots on the basis of the stratum associated with the prediction of proportion forest area for the pixel corresponding to the centre of the central subplot. This is the most simple approach and allows the map's pixel predictions to be used directly to stratify plots. However, this approach assumes that the spectral values of the single pixel associated with the centre of the central subplot adequately characterize the entire plot. The second approach does not require this assumption and assigns plots to strata on the basis of the mean of the predictions of proportion forest area for the four pixels corresponding to the four subplots. This approach assumes that errors in spatial locations of the subplots relative to the image pixels are small and that pixels corresponding to the centres of the four subplots are always in the same geographic configuration. A third approach is less sensitive to violations of the previous assumptions and assigns plots to strata on the basis of the mean of proportion forest area predictions for a 3x3 block of pixels with centre pixel corresponding to centre of the central subplot. Strata weights are calculated by assigning each pixel to a stratum on the basis of the mean of proportion forest area predictions for the 3x3 block of pixels with centre at that pixel. For this study, the latter 3x3 approach was selected.

For each prediction technique and each of the corresponding five best TM band combinations for each study area, four optimal strata were selected where optimality was with respect to maximizing relative efficiency, \( \text{RE} \), the ratio of the variance of the estimated mean obtained using simple random analyses and the variance of the estimated mean using stratified estimation. From the continuum of predictions for each band combination, the four optimal strata were selected from among all possible divisions of the continuum into four classes or intervals under three constraints: first, the lower bound of the first stratum was always 0.00, and the upper bound of the fourth stratum was always 1.00; second, the minimum stratum width was 0.05; and third, at least five plots were required to be assigned to each stratum. Stratifications were limited to four strata, because the preponderance of proportion forest area observations were either 0.00 or 1.00.

Stratified estimates of mean plot proportion forest area, \( \bar{Y} \) and estimated variance, \( \text{Var} (\bar{Y}) \), were calculated using standard methods (Cochran 1977):

\[
\bar{Y} = \sum_{j=1}^{J} w_j \bar{Y}_j
\]

(6)

and

\[
\text{Var}(\bar{Y}) = \sum_{j=1}^{J} w_j^2 \hat{\sigma}_j^2 / n_j
\]

(7)

where \( j = 1, \ldots, J \) denotes stratum; \( w_j \) is the weight for the \( j^{th} \) stratum, calculated as the proportion of pixels assigned to the stratum; \( \bar{Y}_j \) is the mean proportion forest area for plots assigned to the \( j^{th} \) stratum; \( n_j \) is the number of plots assigned to the \( j^{th} \) stratum; and \( \hat{\sigma}_j^2 \) is the within-stratum variance for the \( j^{th} \) stratum calculated as,

\[
\hat{\sigma}_j^2 = \frac{1}{n_j - 1} \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_j)^2
\]

(8)

where \( Y_{ij} \) is the proportion forest area observed by the field crew for the \( i^{th} \) plot in the \( j^{th} \) stratum. Variance estimates obtained using (7) ignore the slight effects due to finite population correction factors and to variable rather than fixed numbers of plots per strata.
3.4 Comparisons

For each study area, the estimate of the mean and the standard error of the mean were calculated under the assumption of simple random sampling for comparison purposes and are denoted SRS. For each prediction technique and for each of the five best band combinations for each study area, the stratified estimates of the mean and standard error of the mean and RE were calculated.

The National Land Cover Dataset (NLCD) (Vogelmann et al. 2001) has also been investigated by several regional FIA programs as a basis for creating strata for stratified estimation. The NLCD, a digital product of the Multi-Resolution Land Characterization (MRLC) Consortium (Loveland and Shaw 1996), is a 21-class, 30m x 30m pixel-based, land cover map of the conterminous United States based on nominal 1992 Landsat 5 Thematic Mapper (TM) satellite imagery and a variety of ancillary data. McRoberts et al. (2002) investigated the utility of the NLCD for variance reduction purposes and recommended creating four strata using a three-step process: (1) aggregate NLCD classes 33 (transitional), 41 (deciduous forest), 42 (evergreen forest), 43 (mixed forest), 51 (shrubland), and 91 (woody wetland) into a forest stratum, and the remaining classes into a non-forest stratum; (2) reclassify isolated groups of three or fewer pixels into their surrounding class to accommodate FIA forest land definitions; and (3) create a forest edge stratum by removing from the forest stratum a 2-pixel wide band on the forest side of the forest/non-forest boundary and create a non-forest edge stratum by removing from the non-forest stratum a 2-pixel wide band on the non-forest side of forest/non-forest boundary. These four strata were created for both study areas using the NLCD, and stratified estimates of the mean and the standard error of the mean and RE were calculated for comparison purposes.

4.0 RESULTS

4.1 Logistic model

Implementation of the logistic modeling approach to creating maps of proportion forest area was straightforward and quick. Use of linear regression for fitting the transformed model, (2), made selection of the best band combinations for each study area quick and easy and also provided initial parameter estimates for the nonlinear regressions. After the five best bands had been selected for each study area and the corresponding nonlinear model parameters had been estimated, the pixel prediction phase was also quickly accomplished.

The five best band combinations within study areas were similar (Table 1). For both study areas, the five band combinations with smallest RMSe, all included five bands. For the St. Louis study area, all five best band combinations included M4 and M5, while four combinations included N3 and N5; for the St. Cloud study area, all five best band combinations included J2, M3, and M4. For both study areas, the spring bands (May for the St. Louis study area; March for the St. Cloud study area) were selected most frequently. The models explained well over half the uncertainty in the subplot observations of forest area proportions, approximately 58 percent for the St. Louis study area and approximately 67 percent for the St. Cloud study area. However, given that forest area proportions were constrained to the interval [0,1], the RMSe values of approximately 0.29 for the St. Louis study area and approximately 0.24 for the St. Cloud study area were relatively large.

4.2 k-NN

Calibration of the k-NN technique was straightforward but more time consuming that the logistic model technique. The five best band combinations were similar in that bands 3 and 4 were most frequently selected for both study areas and that RMSe values were similar (Table 1). The combinations were dissimilar in that knock was considerably larger for the St. Cloud study area than for the St. Louis study area and that different numbers of bands were selected among the five best combinations. The k-NN predictions explained about the same proportion of uncertainty as did the logistic models, approximately 61 percent for the St. Louis study area and approximately 67 percent for the St. Cloud study area.
4.3 Comparisons

Within study areas, estimates of mean plot proportion forest area were similar for the SRS, the NLCD, the five logistic model combinations, and the five k-NN combinations (Table 2). However, the estimates of the standard errors of the means were quite different. The estimates of standard errors for the NLCD, logistic model, and k-NN estimates were all considerably smaller than the SRS estimates, indicating that the maps substantially contributed to reducing the variance and increasing the precision of the estimates of mean forest area proportion. For the St. Louis study area, the five logistic model estimates of the standard errors were similar and the five k-NN estimates were similar, although the logistic model estimates were slightly smaller than the k-NN estimates. For the St. Cloud study area, the five logistic model estimates were again similar as were the five k-NN estimates, but the k-NN estimates were much smaller than the logistic model estimates.

For both the logistic model and k-NN approaches and for both study areas, the ranking of the five best band combinations with respect to RMS<sub>e</sub> obtained when calibrating the techniques was slightly different than the ranking with respect to the stratified estimates of the standard errors of the means. Thus, if an optimal stratification is desired, it is appropriate to consider several of the best band combinations as was done for this study.

Table 2. Stratified estimates of proportion forest area using maps as the basis for stratifications.

<table>
<thead>
<tr>
<th>Combination</th>
<th>Bands</th>
<th>Logistic model</th>
<th>k-NN</th>
<th>RMS&lt;sub&gt;e&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>St. Louis study area</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>N3 N5 M2 M4 M5</td>
<td>0.2854</td>
<td>N3 N4 M4</td>
<td>0.2652</td>
</tr>
<tr>
<td>2</td>
<td>N3 N5 M1 M4 M5</td>
<td>0.2865</td>
<td>N1 N3 M4</td>
<td>0.2687</td>
</tr>
<tr>
<td>3</td>
<td>N2 N5 M1 M4 M5</td>
<td>0.2862</td>
<td>N2 N3 M4</td>
<td>0.2690</td>
</tr>
<tr>
<td>4</td>
<td>N3 N6 M1 M4 M5</td>
<td>0.2875</td>
<td>N3 N4 M1 M4</td>
<td>0.2690</td>
</tr>
<tr>
<td>5</td>
<td>N3 N5 M3 M4 M5</td>
<td>0.2925</td>
<td>N2 N3 N4 M1 M4</td>
<td>0.2693</td>
</tr>
</tbody>
</table>

| St. Cloud study area |      |                |      |                |
| 1          | J2 J5 O7 M3 M4 | 0.2324 | J2 J3 M3 M4 | 0.2392 |
| 2          | J2 J5 O5 M3 M4 | 0.2342 | J2 J3 M1 M3 M4 | 0.2399 |
| 3          | J2 O4 O7 M3 M4 | 0.2387 | J2 J3 M2 M3 M4 | 0.2406 |
| 4          | J2 O3 M1 M3 M4 | 0.2394 | J3 M3 M4 | 0.2420 |
| 5          | J2 O2 M1 M3 M4 | 0.2402 | J3 M1 M3 M4 | 0.2423 |

The proportion forest area maps for the 15 km x 15 km centre of the St. Louis study area obtained using the logistic model with minimum RMS<sub>e</sub> and the k-NN combination with minimum RMS<sub>e</sub> both portrayed a substantial portion of the forest/non-forest detail when compared to a 1992 aerial photograph and the
corresponding NLCD map for the same area (Figure 2). The water is clearly identified as is much of the road network, the airfield in the lower left quadrant, non-forested areas in the upper left quadrant, and a large area also in the upper left quadrant that was apparently cleared of vegetation between the date of the aerial photograph and the date of the TM imagery.

5. CONCLUSIONS

Two primary conclusions may be drawn from this study. First, maps of proportion forest area may be quickly and easily obtained using TM imagery, inventory plot data, and either the logistic model or the k-NN approach. Second, the maps were sufficiently accurate to produce stratifications that reduced the variances of the estimates by factors as great as 6 for the heavily forested area and as great as 5 for the sparsely forested area.

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Figure 2. Forest/non-forest and proportion forest area maps.

- 1992 aerial photograph
- 1992 NLCD forest/non-forest map
- 1999-2000 logistic model proportion forest area map
- 1999-2000 k-NN proportion forest area map