PER TREE ESTIMATES WITH *N*-TREE DISTANCE SAMPLING: AN APPLICATION TO INCREMENT CORE DATA

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ABSTRACT.—Per tree estimates using the *n* trees nearest a point can be obtained by using a ratio of per unit area estimates from *n*-tree distance sampling. This ratio was used to estimate average age by d.b.h. classes for cottonwood trees (*Populus deltoides* Bartr. ex Marsh.) on the Cimarron National Grassland. Increment cores were sampled from the two cottonwoods closest to a plot center. Within each d.b.h. class, a ratio was obtained with the two-tree estimates of the sum of the sample tree ages in the numerator and the number of trees per acre in the denominator. This ratio estimated average age within the class.

Tree attributes that are time consuming or laborious to obtain, such as increment cores, have often been sampled by selection from a fixed number of trees nearest to a plot center or point located in the field. Per tree estimates such as average age or 10-year increment have then been computed by using simple averages from sample cores (or other attributes), often by d.b.h. classes. Clutter and others (1983) correctly noted that estimation using simple averages with these data is problematic because trees that occupy larger amounts of space are more likely to be selected by this procedure. Thus, trees associated with specified attributes are not sampled in proportion to their frequency in the population. Clutter and others (1983) recommend that increment cores be selected from all trees located within the boundaries of small fixed-sized sample plots. However, the problems associated with selection of increment cores or other attributes from the *n* sample trees nearest to a point can also be discussed within the framework of *n*-tree distance sampling.

N-TREE DISTANCE SAMPLING

N-tree distance sampling is based on selection of the *n* trees closest to a sample point located in the forest population of interest. Plot size is circular based on a radius to the center of the most distant of the *n* trees closest to the point, so that all *n* trees are in the plot (fig. 1). The estimate of trees per acre from *n*-tree sampling with *m* sample points is

$$T_A = \frac{1}{m}\left(\frac{n-1}{n}\right)\sum_{i=1}^{m}\frac{n}{A_i} \qquad (1)$$

where

R_{ni} = distance to tree *n* on plot *i* in feet,

$A_i = \pi \cdot \frac{R_{ni}^2}{43{,}560}$ area of plot *i* in acres,

m = number of *n* - tree plot locations, and

T_A = estimated number of trees per acre.

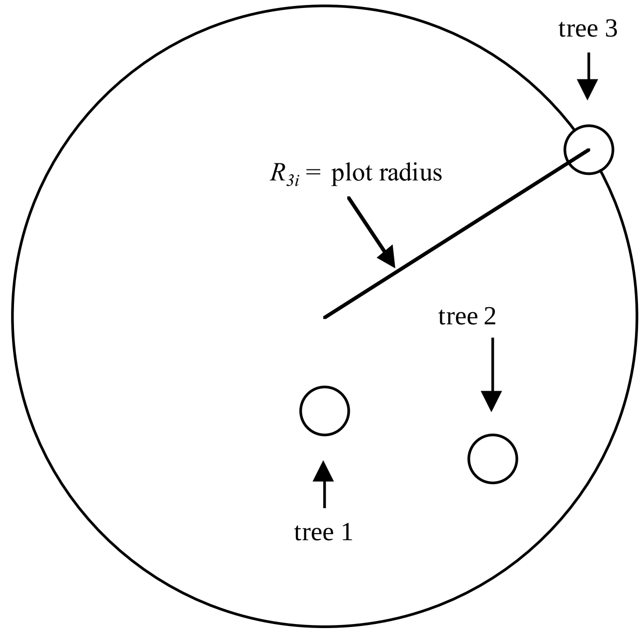

Figure 1.—*Plot for n-tree sampling with n = 3.*

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The estimator above would be similar to a fixed-radius plot estimator except for multiplication of the factor (n-1)/n. Moore (1954) derived this factor to correct for the bias that would otherwise occur when the spatial arrangement of trees is according to the Poisson distribution characteristic of a random pattern. Thompson (1956) derived the distribution of the distance to the n^{th} individual when individuals are randomly distributed on a plane. Eberhart (1967) showed that the use of the factor (n-1)/n is correct for the negative binomial distribution sometimes characteristic of clumped spatial patterns as well as for the Poisson distribution. Per acre estimates of other tree attributes can be obtained by accumulating the attributes of trees on the n-tree sampling plot

$$Y_A = \frac{1}{m}\left(\frac{n-1}{n}\right)\sum_{i=1}^{m}\frac{\sum_{j=1}^{n} Y_{ij}}{A_i} \tag{2}$$

where

Y_{ij} = amount of characteristic Y on tree j at plot i, and

Y_A = estimated amount of characteristic Y per acre.

Lessard and others (1994) used these n-tree methods to estimate volume, basal area, and number of trees per acre in typical northern hardwoods, red pine plantations, and sprout-origin aspen-birch (typically clumped) forest types. They achieved results that compared well with those obtained by more traditional forest inventory methods including fixed-radius plot and point (angle-gauge) sampling.

N-TREE SAMPLING WITH PER TREE ESTIMATES

Estimates of per tree attributes can be obtained by using the ratio of the quantity of an attribute per acre to the number of trees per acre having that attribute. Estimators for the latter quantities can be obtained using the n-tree distance sampling estimators given above. The ratio estimate for per tree attributes from n-tree distance sampling is

$$\overline{Y}_{pt} = \frac{Y_A}{T'_A} = \frac{\frac{1}{m}\left(\frac{n-1}{n}\right)\sum_{i=1}^{m}\frac{\sum_{j=1}^{n} Y_{ij}}{A_i}}{\frac{1}{m}\left(\frac{n-1}{n}\right)\sum_{i=1}^{m}\frac{\sum_{j=1}^{n} I_{ij}}{A_i}} = \frac{\sum_{i=1}^{m}\frac{\sum_{j=1}^{n} Y_{ij}}{A_i}}{\sum_{i=1}^{m}\frac{\sum_{j=1}^{n} I_{ij}}{A_i}} \tag{3}$$

where

I_{ij} is 1 if tree j on plot i has attribute Y, otherwise 0,

T'_A is the estimated number of trees per acre having attribute Y, and

$\overline{Y}_{pt}$ is the estimate for the average amount of attribute Y per tree.

It is of interest to note that the ratio estimate $\overline{Y}_{pt}$ does not depend on the bias correction factor (n-1)/n since it is eliminated by division of per acre estimates when the ratio is formed. This suggests $\overline{Y}_{pt}$ may be robust in application to forests that have spatial arrangements for which the bias correction factor (n-1)/n may not be correct.

ESTIMATING AVERAGE TREE AGE ON THE CIMARRON NATIONAL GRASSLAND

The ratio estimate of equation 3 above was used to estimate the average age per tree in a recent inventory of the tree resources of the Cimarron National Grassland, located in southwestern Kansas. Eastern cottonwood is the predominant species making up the riparian forest along a 25-mile-long corridor surrounding the Cimarron River within the grassland. Nineteen sample transects crossing the riparian area surrounding the river from north to south were established at points randomly located from the western border of the grassland. The band of trees in the river corridor is surrounded by semi-arid grassland. Each transect covered the width of the tree band. Fixed-radius 0.1-acre (0.2 acre on the first two transects) sample plots were established at 6-chain (396 feet) intervals along each transect on which sample trees were selected to estimate the number of trees per acre and basal area per acre. Thus, the number of plots per transect varied according to the width of the riparian forest. This was an unequal-sized cluster sampling design (Sukhatme and others 1984) in which the transects consisted of clusters of plots. Two cottonwood trees (three on the first two transects) nearest each plot center were selected for increment core extraction to determine tree age. Cores were collected and aged in the laboratory where phloroglucinol staining was used to enhance separation between springwood and summerwood as recommended by Patterson (1959) for diffuse porous species. Cottonwood trees ranging from seedlings to trees 47 inches in d.b.h. were sampled. On average, 94 cottonwood trees per acre were greater than 4.5 feet tall, of which 34 per acre were greater than 3.5 inches in d.b.h.

Equation 3 was used to estimate average age per tree within 1-inch d.b.h. classes for cottonwood on the Cimarron National Grassland. Standard errors were estimated using methods appropriate for the ratio of sample means (Cochran 1977). For comparison, ages within d.b.h. classes were also calculated by a simple arithmetic average of increment cores within the selected class.

Table 1.—*Estimates of average age for selected 1-inch d.b.h. classes on the Cimarron National Grassland from a ratio of n-tree sampling estimates and from unweighted arithmetic averages of increment cores*

D.b.h. class midpoint (inches)	Ratio estimate of average age (yrs) (standard error)	Unweighted average age (yrs) (standard error)
3	12.0(0.1)	13.2(1.2)
16	60.4(4.3)	46.7(3.9)
31	79.0(3.4)	79.6(2.8)

Table 1 indicates that the two methods of average age estimation give results that are quite close in the 3-inch and 31-inch classes. However, estimates could be substantially different, as was the case in the 16-inch d.b.h. class where the greatest difference in age estimates was observed.

Inspection of the data in the 16-inch d.b.h. class indicated that one of the older cores was extracted from a tree located on an unusually small *n*-tree plot. Cottonwood tends to grow in clumps, so if the sample point falls in a clump of two or more trees, it is possible to obtain a rather small plot in two-tree sampling. Equation 3 is similar to a weighted average of increment core data where the weights are the inverse of the *n*-tree plot sizes. Thus, observations associated with small plot sizes have more weight than those associated with larger plot sizes. Therefore, in the 16-inch classes, the older core received more weight and caused the resulting average age estimate to be larger than the simple arithmetic average of the core data. Of course, this weighting is usually desirable and helps to overcome the objections of Clutter and others (1982) to the use of nearest-neighbor tree data for per tree estimation. There is a tendency for trees on smaller plots to grow in higher densities and therefore to have smaller probability of selection by nearest-neighbor methods. Weighting by inverse plot size helps to account for this when data are used in estimators such as equation 3.

SUMMARY AND DISCUSSION

Generally it should be desirable to use equation 3 to estimate per tree attributes when the *n* nearest trees are selected as sample trees. This method helps avoid the objections of Clutter and others (1983). In the context of *n*-tree sampling for per hectare attributes, Jonsson and others (1992) recommended sampling all trees within a small fixed-radius plot in cases where the *n*-tree plot radius was smaller than a given fixed amount (they used 3 m for 6-tree to 13-tree sampling). However, Jonsson and others (1992) did not discuss estimation of per tree attributes. A ratio of per acre estimates from *n*-tree sampling was used to estimate average ages by d.b.h. class on the Cimarron National Grassland. In most d.b.h. classes, these estimates were not greatly different from simple arithmetic averages of increment core data, but, there were fairly substantial differences in a few classes. The estimate of average age given by the ratio of per acre *n*-tree estimates should be preferable since it accounts for the variation in *n*-tree plot sizes and corresponding fluctuations of density within the tree population.

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