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Growth and Yield Model for the Elm-Ash-Cottonwood Type in Indiana

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GROWTH AND YIELD MODEL FOR THE ELM-ASH-COTTONWOOD TYPE IN INDIANA

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More than 0.5 million acres of commercial elm-ash-cottonwood forest can be found in Indiana (Spencer 1969) with an additional 1.4 million acres occurring in Illinois (Essex and Gansner 1965). The percent of commercial elm-ash-cottonwood forest land in these two States is surpassed only by that of oak-hickory. Most elm-ash-cottonwood stands occur on good to excellent sites. Despite its abundance, however, little has been published concerning the growth and yield of the elm-ash-cottonwood (E-A-C) forest type in the Central States (Shifley and Brown 1978).

This paper reports the results of a recent study of the growth and yield of the E-A-C forest type in Indiana. The study led to the development of a system of mathematical models that have been used to generate yield tables for E-A-C stands.

TYPE DEFINITION

The E-A-C forest type is defined here as any bottomland forest in which elm (*Ulmus* spp.), green ash (*Fraxinus pennsylvanica* Marsh.), cottonwood (*Populus deltoides* Marsh.), silver maple (*Acer saccharinum* L.), or red maple (*Acer rubrum* L.), singly or in any combination, comprise a plurality of the stocking. Willow (*Salix nigra* Marsh.), boxelder (*Acer negundo* L.), and sycamore (*Platanus occidentalis* L.) are common associates. Although the E-A-C species mixture inhabits a variety of poorly drained upland sites, only occurrences of the type adjacent to waterways were considered in this study.

DATA

Remeasurement data were collected in 1977 from 35 continuous forest inventory (CFI) sampling units located in Indiana E-A-C stands of natural origin.[1] Each sampling unit was comprised of 10 subsamples equally spaced over approximately 1 circular acre. At each subsample point trees greater than or equal to 5 inches d.b.h. were sampled using a 37.5 factor prism. Trees between 1 and 5 inches d.b.h. were sampled on a 1/300 acre circular plot centered on the prism point. Annual ingrowth, survivor growth, mortality, and cut were estimated on a per-acre basis for each of the sampling units (table 1). These annual rates of change were used to construct the rate equations in subsequent sections.

Although pure stands of even-aged cottonwood, willow, or mixtures of these two species are classic early stages in E-A-C succession on newly-formed land, the stands encountered in the data collected for this study are characterized by mixed species and variable size classes as found in subsequent seral stages or in E-A-C development on old field sites (Hosner and Minkler 1963).

PAST WORK

Whole-stand, age-independent modeling techniques are well suited to predict E-A-C stand dynamics because such models can be calibrated with few data and for both even-aged and uneven-aged stands.

MacKinney *et al.* (1937) and Schumacher (1939) present some of the earliest mathematical forest stand growth and yield models. It was not until more recently, however, that Buckman (1962) and Clutter

[1]*Data from Indiana State Survey were provided by the Renewable Resources Evaluation Project, North Central Forest Experiment Station.*

(1963) demonstrated that differential equations could be applied to obtain compatible growth and yield estimates for even-aged forest stands. Pienaar (1965) and Turnbull (1963) examine in detail the biomathematical aspects of growth and yield estimation in pure and mixed stands. Pienaar and Turnbull (1973) emphasize the importance of biologically reasonable growth and yield models, and they suggest the Chapman-Richards function as a suitable model for a variety of even-aged stand growth applications. While previous attempts to model stand growth using differential equations had been limited to even-aged stands, Moser and Hall (1969) extended differential equation theory to applications involving growth and yield in uneven-aged stands.

Leary (1970) suggests that systems for modeling forest stand dynamics may be separated into components, each component defined by one or more difference equations within a system of equations which, in total, describe stand growth. In such a system stand yield is determined through the simultaneous solution of the component equations. Leary gives an example in which tree size classes are the components through which stand dynamics are modeled, but he suggests that the components used to model stand dynamics will vary with the application. Moser (1972) presents a system of differential equations describing uneven-aged stand growth in which the components used are ingrowth, survivor growth, and mortality in the stand, i.e., the components of growth (Beers 1962).

Both Moser (1974) and Ek (1974) have demonstrated that more complex stand modeling systems are feasible. In separate studies involving applications with uneven-aged mixed hardwoods, each author models stand growth by dividing the stand into a number of component size classes. Growth within each size class is further separated into expressions for ingrowth, survival growth, mortality, and upgrowth. Moser uses a system of 66 equations (11 equations for each of 6 size classes) to directly simulate change in number of trees, basal area, and cubic foot volume. Ek achieves a similar goal with fewer equations by simulating change in the number of trees by 2-inch diameter classes and then estimating basal area and volume from the known number of trees in each diameter class.

Markov models offer an alternative to stand growth simulation by differential equations (Peden *et al.* 1973, Bruner and Moser 1973). Although Markov models characterizing stand dynamics may serve many useful purposes, they generally do not provide the same opportunities for theorization that are offered by differential equation-based models.

THE ELM-ASH-COTTONWOOD MODEL

A system of differential[2] equations was sought that would simulate change in the number of trees and basal area of E-A-C stands. Ideally, such a model should be the simplest one that describes the biological phenomena (Milsum 1966) and remains consistent with the structure and function of the actual biological system (Pienaar 1965). The methods used here are similar to those of Moser (1972) that divide stand growth into the components of ingrowth, mortality, and survivor growth. Modeling stand growth by diameter classes was considered infeasible due to the small amount of data available to calibrate the model.

The following notation will be used to describe the model.

dN_i/dt = Rate at which ingrowth trees cross the 5-inch d.b.h. threshold (units: trees/acre/year).

dN_m/dt = Rate at which trees 5 inches d.b.h. or larger die (units: trees/acre/year).

N = Total number of trees 5 inches d.b.h. or larger (units: trees/acre).

dB_i/dt = Basal area represented by the ingrowth trees (units: square feet/acre/year).

dB_m/dt = Basal area represented by the mortality trees (units: square feet/acre/year).

dB_s/dt = Basal area growth on the surviving N trees (units: square feet/acre/year).

B = Total basal area of the N trees (units: square feet/acre).

The net rate of change in the number of trees per acre per year can be analyzed as the algebraic sum of the ingrowth and mortality rates (Moser 1972):

$$\frac{dN}{dt} = \frac{dN_i}{dt} - \frac{dN_m}{dt} \qquad (1)$$

The net rate of change in basal area per acre per year can be analyzed similarly. Its components are the rates of change due to ingrowth, mortality, and survivor growth:

$$\frac{dB}{dt} = \frac{dB_i}{dt} - \frac{dB_m}{dt} + \frac{dB_s}{dt} \qquad (2)$$

[2] *Although a difference equation formulation is theoretically appropriate, we favored differential equations for analytical convenience. Numerical differences in solutions due to this choice are small.*

Table 1.—*Initial distribution of calibration data by basal area and number of trees per acre (trees 5 inches d.b.h. and larger)*

Basal area (ft^2/acre)	Trees (No./acre) 20	40	60	80	100	120	140	160	180	200	220	240
10	2											
20		1	1									
30					1							
40			1	1		1						
50		1			1		1					
60					4	1	1	1				
70						1		2				
80				1		2	1		3			
90						2					1	
100									1			
110									1			
120												
130												1
140								1				

Equations (1) and (2) form the basis for a system of interactive differential equations that describe forest stand dynamics. To be of practical value, however, the derivatives on the right-hand sides of Equations (1) and (2) must be replaced by differential equations that are both biologically plausible and mathematically tractable. The development of these equations is presented in the following sections.

Ingrowth Rate: Number of Trees

Ingrowth occurs when trees enter the smallest measured diameter class from below. In this case a tree is classified as an ingrowth tree when it reaches 5 inches d.b.h. In real forest stands, ingrowth is a discrete phenomenon—an all or none event. However, for the stand model being developed here, ingrowth is treated as if it were a continuous process. Over the long run, the model predicts reasonable numbers of ingrowth trees.

Studies have indicated that stand density is a major factor influencing the number of seedlings and saplings in a stand. Minckler (1958) reports an increase in the number of saplings (0.6 to 4.6 inches d.b.h.) after E-A-C stand basal area was reduced by 32, 63, and 100 percent. Belanger and Pepper (1978) report a significant decrease in diameter growth of sycamore seedlings as number of trees per acre increase.

Because E-A-C stands are characteristically comprised of species that are intermediately tolerant or intolerant of competition, it is hypothesized that the rate of ingrowth can be characterized as a function of stand density. Both Ek (1974) and Moser (1972) have used negative exponential functions to express the periodic rate of ingrowth as a function of stand density. Examination of our E-A-C data showed that the annual rate of ingrowth decreased as basal area increased and that the rate of ingrowth was higher in stands with a small average diameter. These relations can be expressed as:

$$\frac{dN_i}{dt} = 10.14e^{(0.002456N\ -\ 0.01560B)} \qquad (3)$$

MSE (Mean Square Error) = 14.6
e = base of natural logarithms.

The greatest annual rates of ingrowth are predicted to occur in stands with large numbers of small diameter trees. According to Equation (3), if all of the trees in a stand are less than 5 inches d.b.h., (N and B both equal 0), the ingrowth rate would be about 10 trees per acre per year. Actual forest stands containing no trees more than 5 inches d.b.h. can range from bare ground to stands with thousands of saplings per acre. Such widely different stand conditions would produce different numbers of ingrowth trees. Therefore, Equation (3) should not be used to estimate ingrowth in stands where most of the trees are less than 5 inches d.b.h.

Mortality Rate: Number of Trees

Numerous authors have attempted to predict individual tree mortality, and they have found that the mortality rate decreases as tree size increases, growth rate increases, vigor improves, or competition decreases (Monserud 1976, Hamilton and Edwards 1976, Buchman 1979). In stand simulation models, the number of mortality trees has been expressed variously in functions that predict increases in mortality with increases in number of trees and/or basal area (Ek 1974, Moser 1974). Lee (1971) related mortality in even-aged stands to stand age and mean stand diameter.

The annual number of E-A-C mortality trees was positively correlated with density as measured by the number of trees and the basal area per acre. Annual mortality was expressed as separate functions of basal area and number of trees but several models combining these two independent variables into one function did not improve statistical accuracy or biological reasonableness. Consequently, the following model was judged a suitable expression for the annual number of E-A-C mortality trees per acre.

$$\frac{dN_m}{dt} = 1.106e^{\text{[illegible]}} \qquad (4)$$

$$MSE = 2.7$$

Ingrowth Rate: Basal Area

The rate of basal area increase due to ingrowth is a direct function of the rate at which ingrowth trees occur. A 5-inch d.b.h. tree contributes 0.1364 square foot of basal area, so the relation between dN_i/dt and dB_i/dt is given by the differential equation:

$$\frac{dB_i}{dt} = 0.1364 \frac{dN_i}{dt} \qquad (5)$$

Because ingrowth is treated as a continuous process in the solution of this system of equations, that portion of the remeasured ingrowth trees' basal area in excess of 0.1364 square foot per tree was treated as survivor growth when the E-A-C data were analyzed.

Mortality: Basal Area

Mortality trees occur in all diameter classes, so a single large mortality tree may account for the same basal area as several smaller ones. Moser (1972) estimated the basal area of mortality trees by specifying a diameter distribution and assigning an appropriate diameter to each projected mortality tree.

Bailey and Dell (1973) used the Weibull probability density function to describe diameter distributions similar to those observed for E-A-C mortality trees. When properly calibrated, the cumulative form of the Weibull frequency distribution:

$$x = a + b\,[-\ln(1 - F(x))]^{c^{-1}} \qquad (7)$$

where

x = the tree diameter
a = location parameter
b = scale parameter
c = shape parameter

can be used to generate diameters of mortality trees if $F(x)$ is replaced by a uniformly distributed random variable. The maximum likelihood parameter estimates of the distribution of mortality trees by diameter class, obtained by the method of Harter and Moore (1965), were $a = 5.0$, $b = 3.142$, and $c = 0.9057$. A Komolgorov-Smirnov test supported the null hypothesis that the fitted Weibull distribution is equivalent to the observed distribution at the $\alpha = 0.1$ significance level. Consequently, the function

$$\frac{dB_m}{dt} = [5.0 + 3.142(-\ln(1-R))^{1.104}]^2 \; 0.005454 \; \frac{dN_m}{dt} \qquad (8)$$

where

R = a uniformly distributed random variable

generates a new basal area mortality estimate for each iteration in the solution of the system of equations.

This method of determining mortality basal area cannot be expected to perform well for stands that are very different from those from which the diameter distribution of mortality trees was obtained. The mean diameter for the remeasured stands was estimated to be between 8.5 and 14.2 inches at the 0.9 probability level. For the instances in which a simulated stand has a mean diameter of less than 8.5 inches or more than 14.2 inches, the mortality trees are assumed to be of average size for that stand. This avoids predicting unreasonably large or small mortality tree diameters.

Survivor Growth: Basal Area

The amount of basal area survivor growth is expected to depend upon the size and number of the survivor trees. Moser (1972) described stand basal area survivor growth using the Chapman-Richards function with stand basal area as the independent variable. Although stand basal area was correlated with the growth rate of E-A-C survivor trees, the best results were obtained by relating the basal area survivor growth to the stand sum of diameters. The sum of diameters for the stand was approximated as the product of the number of trees in the stand and the quadratic mean stand diameter, $N\,(B/(N \cdot .005454))^{1/2}$. Linear regression through the origin was used to obtain the expression for annual basal area growth of survivor trees,

$$\frac{dB_s}{dt} = 0.03043\,(B \cdot N)^{1/2} \qquad (6)$$

$$R^2 = .82^{**}$$

$$MSE = 1.7$$

Although this function states that survivor basal area growth increases proportionately for all increases in the sum of diameters, simulated stand growth does have an upper limit. As the basal area increases, stand mortality also increases to limit basal area growth.

RESULTS AND DISCUSSION

The entire system of differential equations describing the rates of change in number of trees and basal area for elm-ash-cottonwood stands is as follows:

$$\frac{dN_i}{dt} = 10.14e^{(0.002456N - 0.01560B)}$$

$$\frac{dN_m}{dt} = 1.106e^{0.006561B}$$

$$\frac{dN}{dt} = \frac{dN_i}{dt} - \frac{dN_m}{dt}$$

$$\frac{dB_i}{dt} = 0.1364 \frac{dN_i}{dt}$$

$$\frac{dB_s}{dt} = 0.03043 (B \cdot N)^{1/2}$$

$$\frac{dB_m}{dt} = [5.0 + 3.142(-1n(1-R))^{1.104}]^2$$

$$0.005454 \frac{dN_m}{dt}$$

$$\text{if } 0.40 \langle \frac{B}{N} \langle 1.10$$

$$= \frac{B}{N}\frac{dN_m}{dt} \text{ otherwise}$$

$$\frac{dB}{dt} = \frac{dB_i}{dt} - \frac{dB_m}{dt} + \frac{dB_s}{dt}$$

The above system of equations was utilized with a fourth order Runge-Kutta algorithm (International Business Machines 1968) to develop a simulator that generates E-A-C growth and yield estimates in basal area and number of trees per acre. Inputs to the simulator are the initial values for basal area and number of trees. Outputs are the projected values for basal area and number of trees by growth components at user-specified time intervals. Tables 4 through 15 in Appendix I contain 25-year projections of total basal area, total number of trees, and cubic foot volume excluding bark for trees 5 inches d.b.h. and larger. Volume estimates were derived from the projected stand basal area using the following formula:

$$V = 8.319B^{1.211} \qquad (9)$$

where

V = cubic foot volume excluding bark for trees $\geqq$ 5 inches d.b.h. and
B = basal area.

This function was fitted from the initial measured basal area and cubic foot volume in the E-A-C data. Tree volumes were estimated from the composite hardwood volume table formulae prepared by Beers (1964).

The yield tables in Appendix I are based on a single simulation trial. Because the basal area mortality function Equation (7) is stochastic, it will permit some variation in the final predictions. However, the simulation that produced the Appendix tables was performed in a manner such that from 50 to several hundred iterations were made during the solution process. A different-sized mortality tree was selected for each iteration and these small, numerous iterations made the effects of using a particular sequence of random numbers to specify mortality tree diameters almost negligible. Repeated trials with different sequences of random numbers produced results that varied by only 1 or 2 square feet of basal area after 25 years of simulation. However, it remains possible to alter the simulation program so that basal area mortality produces greater random variation in the basal area mortality during simulation trials. The FORTRAN program used for simulation is listed in Appendix II.

No data were reserved for independent validation and testing. Tests were conducted, however, to determine how well the model could predict the observed changes in stand characteristics for the 11-year remeasurement interval of the calibration data (table 2, fig. 1). Minckler's (1958) bottomland hardwood cutting study produced a small number of CFI plots suitable for independently validating the E-A-C model.[3] The study included three control plots, each approximately 2 acres in size. These plots were established in 1953 and last remeasured in 1968. The simulator was used to project the initial conditions for each of these plots to an estimate of their 1968 condition (table 3). Individual differences in the observed and predicted basal areas for the three plots after 15 years were +2, +12, and-19 ft²/acre. The greatest error occurred on a plot within an even-aged cottonwood stand.

A desirable property of any growth simulation model is that it gives reasonable results for long projection periods. There are no data to evaluate the

[3]*Data provided by Richard Schlesinger, North Central Forest Experiment Station, Carbondale, Illinois.*

Table 2.—*Eleven-year mean observed and mean predicted values for 35 elm-ash-cottonwood calibration plots (trees 5 inches d.b.h. and larger)*

	Trees	Basal area	Volume
	No./acre	*Ft²/acre*	*Ft³/acre*
Initial	109	62	[1]1,270
Observed	152	90	[1]2,053
Predicted	137	92	[2]2,017
Difference	-15	+2	-36
Std. error of difference	9	3.4	101

[1]Computed from the sum of observed individual tree volumes.
[2]Computed as a function of basal area, Equation (9).

accuracy of long-term E-A-C projections, but one may be encouraged if estimated stand conditions remain believable for a long projection period. Projections of two E-A-C stands for 150 years produced reasonable volume estimates (fig. 2). Although long-range projections provide an enlightening test of a model's capabilities, it is seldom prudent to rely on projections that are much longer than the interval represented by calibration data.

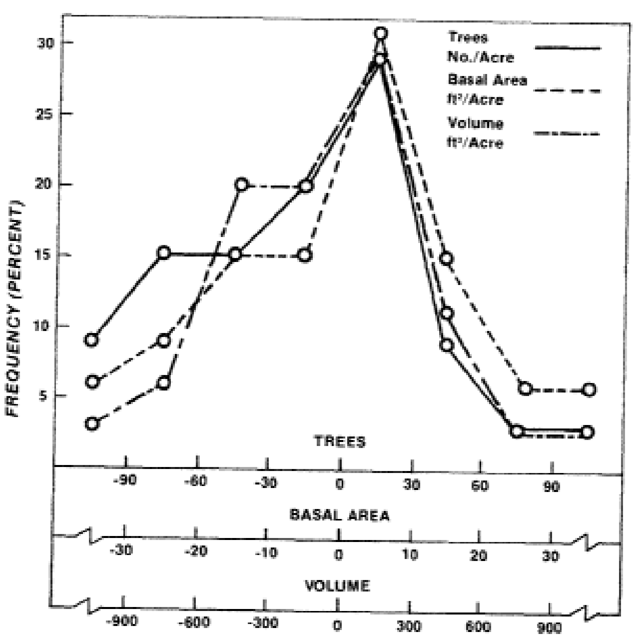

Figure 1.—*Frequency of errors in 11-year projections of 35 elm-ash-cottonwood calibration plots.*

Table 3.—*Fifteen-year mean observed and predicted values of trees 5 inches d.b.h. and larger for three elm-ash-cottonwood validation plots*

Item	Trees	Basal area	Volume
	No./acre	*ft²acre*	*ft³/acre*[1]
Initial	89	89	1,927
Final observed	83	114	2,600
Final predicted	96	113	2,533
Difference	+13	-1	-67

[1]Volumes computed as a function of basal area, Equation (9).

The growth models and simulated yield tables presented here outline some general patterns of E-A-C stand development in Indiana's bottomland. Additional research is necessary to document the effects of species composition, age structure, and management activities not addressed in this study but which certainly affect E-A-C stand growth. We hope that the information presented here will facilitate greater understanding and better utilization of the elm-ash-cottonwood forest resource.

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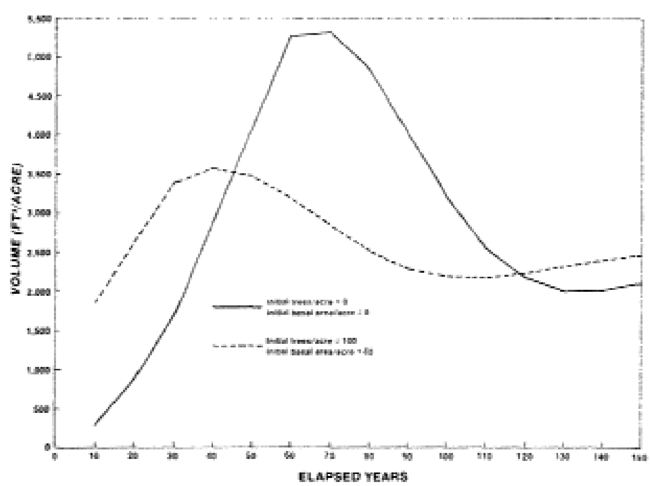

Figure 2.—*Projected volume of trees 5 inches d.b.h. and larger on two elm-ash-cottonwood plots.*

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APPENDIX I

SIMULATED ELM-ASH-COTTONWOOD YIELDS

Table 4.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 20 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
10	135	[1]59	96	128	155	175
		18	28	41	55	72
		268	471	740	1,074	1,466
20	313	52	82	109	130	147
		27	36	47	60	74
		446	637	878	1,177	1,532
30	512	47	71	93	110	123
		35	44	54	66	79
		619	810	1,044	1,329	1,658

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 5.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 40 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
		[1]81	118	150	175	193
10	135	19	31	46	62	79
		303	541	850	1,226	1,661
		74	104	130	150	165
20	313	28	39	52	66	82
		479	709	997	1,335	1,730
		67	91	111	127	138
30	512	38	48	60	73	88
		685	911	1,182	1,505	1,874
		62	80	96	107	114
40	725	48	58	68	81	94
		903	1,127	1,389	1,695	2,034
		58	73	84	93	98
50	950	56	65	75	87	99
		1,089	1,306	1,563	1,852	2,165
		54	67	77	84	89
60	1,185	61	69	79	90	98
		1,199	1,404	1,649	1,928	2,138

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 6.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 60 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
		[1]103	141	172	196	212
10	135	21	34	50	68	87
		330	602	950	1,370	1,849
		95	126	151	170	182
20	313	30	43	57	73	90
		518	787	1,118	1,507	1,948
		89	113	133	148	157
30	512	40	51	64	79	95
		716	976	1,286	1,651	2,065
		83	102	117	127	133
40	725	49	61	73	87	101
		938	1,201	1,508	1,859	2,240
		78	92	102	109	112
50	950	60	71	82	95	108
		1,717	1,442	1,738	2,061	2,421
		73	84	91	94	95
60	1,185	69	80	91	103	108
		1,415	1,674	1,960	2,267	2,412
		70	78	83	88	91
70	1,428	77	86	92	96	101
		1,592	1,827	1,985	2,107	2,225
		68	75	81	86	89
80	1,679	81	84	89	94	99
		1,713	1,787	1,909	2,047	2,164

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 7.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 80 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions: 5	10	15	20	25
		[1]117	148	173	190	201
20	313	32	46	62	79	98
		551	858	1,231	1,664	2,147
		110	135	154	168	176
30	512	41	54	69	85	103
		753	1,052	1,407	1,813	2,268
		104	123	138	147	152
40	725	51	64	78	93	109
		976	1,275	1,623	2,012	2,448
		99	113	123	129	130
50	950	61	73	87	101	116
		1,213	1,515	1,858	2,233	2,640
		94	104	111	113	112
60	1,185	71	83	96	110	124
		1,455	1,760	2,102	2,464	2,845
		90	97	100	100	99
70	1,428	81	93	105	113	117
		1,706	2,010	2,338	2,560	2,674
		87	90	92	93	93
80	1,679	91	100	104	108	112
		1,965	2,198	2,316	2,426	2,523
		85	88	91	92	93
90	1,937	94	98	102	107	110
		2,041	2,153	2,267	2,376	2,475
		82	84	85	85	85
100	2,200	102	104	106	108	110
		2,257	2,316	2,374	2,426	2,472

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 8.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 100 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions: 5	10	15	20	25
		[1]138	171	195	212	221
20	313	33	49	66	85	106
		580	923	1,338	1,816	2,350
		131	157	176	188	194
30	512	43	58	74	92	111
		790	1,130	1,527	1,982	2,485
		125	145	159	168	171
40	725	52	66	82	99	116
		1,003	1,337	1,726	2,166	2,643
		120	135	144	149	150
50	950	63	76	91	107	123
		1,246	1,586	1,967	2,385	2,833
		115	126	132	134	132
60	1,185	72	86	100	115	131
		1,489	1,830	2,208	2,615	3,048
		111	118	120	119	115
70	1,428	83	96	110	125	133
		1,745	2,093	2,473	2,870	3,116
		107	111	111	108	104
80	1,679	93	106	120	126	129
		2,007	2,355	2,735	2,905	3,005
		104	105	103	101	97
90	1,937	103	115	120	124	126
		2,275	2,618	2,747	2,846	2,910
		101	100	99	96	93
100	2,200	112	117	120	123	124
		2,523	2,649	2,748	2,819	2,859
		100	98	96	93	90
110	2,469	115	118	121	123	124
		2,597	2,701	2,778	2,826	2,846
		97	94	91	87	84
120	2,744	123	125	125	125	124
		2,824	2,873	2,890	2,878	2,842

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 9.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 120 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
		[1]160	193	218	233	241
20	313	35	52	71	91	113
		607	985	1,442	1,968	2,550
		153	179	197	209	213
30	512	44	61	79	98	118
		822	1,202	1,644	2,140	2,690
		146	167	180	188	190
40	725	54	70	87	105	124
		1,044	1,422	1,850	2,328	2,863
		141	156	166	170	169
50	950	64	79	95	113	131
		1,277	1,653	2,076	2,539	3,045
		136	147	153	154	151
60	1,185	74	89	105	121	138
		1,523	1,900	2,323	2,780	3,263
		132	139	141	139	134
70	1,428	84	99	115	131	147
		1,782	2,169	2,599	3,049	3,505
		128	132	131	127	120
80	1,679	94	109	124	140	145
		2,046	2,436	2,863	3,298	3,440
		125	125	122	117	110
90	1,937	104	119	134	139	142
		2,317	2,711	3,134	3,282	3,366
		122	120	115	109	103
100	2,200	114	129	135	139	140
		2,592	2,987	3,172	3,267	3,306
		119	115	110	104	97
110	2,469	125	133	137	139	139
		2,872	3,108	3,214	3,267	3,267
		117	112	106	99	93
120	2,744	132	136	138	139	138
		3,072	3,194	3,264	3,280	3,246
		115	109	103	96	90
130	3,023	136	139	140	140	138
		3,195	3,286	3,321	3,303	3,238
		113	106	99	91	85
140	3,307	144	145	144	141	137
		3,408	3,446	3,424	3,350	3,232

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 10.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 140 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
		[1]174	201	219	230	233
30	512	46	64	83	104	126
		852	1,270	1,757	2,301	2,898
		168	188	202	209	209
40	725	56	73	91	111	132
		1,079	1,496	1,965	2,486	3,066
		162	178	187	190	188
50	950	65	82	100	119	139
		1,312	1,723	2,190	2,710	3,268
		157	168	174	174	170
60	1,185	75	91	109	127	146
		1,555	1,971	2,443	2,949	3,481
		152	160	162	159	153
70	1,428	85	102	119	137	155
		1,816	2,244	2,719	3,219	3,729
		149	152	151	146	138
80	1,679	96	112	129	146	158
		2,082	2,529	2,997	3,488	3,842
		145	146	142	135	126
90	1,937	106	122	139	152	157
		2,356	2,795	3,268	3,667	3,795
		142	140	134	126	117
100	2,200	116	132	148	153	155
		2,633	3,075	3,536	3,681	3,749
		139	135	127	119	110
110	2,469	126	142	150	153	154
		2,915	3,363	3,603	3,694	3,710
		137	130	122	113	104
120	2,744	136	148	152	154	153
		3,202	3,547	3,666	3,710	3,680
		135	127	118	108	99
130	3,023	146	152	155	155	152
		3,488	3,660	3,735	3,734	3,661
		133	124	114	104	95
140	3,307	152	156	157	156	152
		3,653	3,756	3,802	3,759	3,648

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 11.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 160 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions 5	10	15	20	25
30	512	[1]196	224	242	251	253
		47	66	87	110	133
		881	1,336	1,866	2,459	3,099
40	725	189	210	223	229	229
		57	76	96	117	139
		1,112	1,369	2,084	2,652	3,282
50	950	183	199	208	211	207
		67	85	104	125	146
		1,348	1,803	2,310	2,874	3,480
60	1,185	178	189	195	194	189
		76	94	113	133	154
		1,590	2,046	2,560	3,119	3,708
70	1,428	173	181	183	179	172
		87	104	123	143	162
		1,849	2,314	2,835	3,384	3,945
80	1,679	169	173	172	166	156
		97	115	133	152	171
		2,117	2,604	3,120	3,661	4,207
90	1,937	166	167	163	155	144
		107	125	143	161	170
		2,393	2,872	3,388	3,918	4,196
100	2,200	163	161	154	144	133
		117	135	153	166	170
		2,672	3,159	3,674	4,063	4,174
110	2,469	160	155	147	136	124
		128	145	162	167	169
		2,956	3,454	3,954	4,099	4,146
120	2,744	157	150	140	129	117
		137	155	165	168	168
		3,235	3,745	4,043	4,131	4,122
130	3,023	155	146	135	123	111
		148	163	168	169	167
		3,533	3,991	4,124	4,158	4,098
140	3,307	153	142	130	118	106
		158	168	171	170	166
		3,826	4,116	4,201	4,186	4,080

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 12.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 180 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions 5	10	15	20	25
40	725	[1]211	232	245	250	249
		58	78	100	122	146
		1,143	1,639	2,198	2,811	3,480
50	950	204	221	229	231	227
		68	88	108	130	153
		1,383	1,881	2,426	3,034	3,686
60	1,185	199	211	216	214	208
		78	97	118	139	161
		1,629	2,125	2,675	3,286	3,916
70	1,428	194	202	204	200	191
		88	107	127	148	169
		1,885	2,384	2,947	3,543	4,162
80	1,679	190	194	193	186	175
		98	117	138	158	178
		2,151	2,672	3,239	3,830	4,428
90	1,937	186	187	183	174	162
		108	127	147	167	183
		2,428	2,949	3,511	4,099	4,577
100	2,200	183	181	174	164	150
		119	138	157	176	183
		2,10	3,239	3,805	4,372	4,573
110	2,469	180	176	167	154	140
		129	148	167	180	183
		2,995	3,538	4,099	4,470	4,561
120	2,744	178	171	160	146	132
		139	158	177	181	182
		3,275	3,833	4,382	4,525	4,532
130	3,023	175	166	153	139	125
		149	169	180	183	182
		3,576	4,142	4,485	4,567	4,550
140	3,307	173	162	148	133	119
		159	178	183	184	181
		3,871	4,429	4,575	4,601	4,513

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 13.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 200 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
		[1]226	242	251	252	246
50	950	69	91	113	136	160
		1,416	1,955	2,552	3,193	3,899
		220	232	237	235	227
60	1,185	79	100	122	145	168
		1,665	2,207	2,795	3,448	4,135
		215	223	224	220	210
70	1,428	89	110	131	154	176
		1,921	2,462	3,062	3,709	4,375
		211	215	213	206	195
80	1,679	99	120	141	163	184
		2,183	2,735	3,333	3,973	4,612
		207	208	203	194	180
90	1,937	110	130	152	173	194
		2,462	3,026	3,640	4,279	4,901
		204	202	195	183	168
100	2,200	120	140	161	182	195
		2,745	3,316	3,929	4,548	4,956
		201	196	187	173	157
110	2,469	130	151	171	191	196
		3,033	3,619	4,228	4,825	4,963
		198	191	179	164	148
120	2,744	140	161	182	194	196
		3,318	3,917	4,532	4,896	4,963
		195	186	172	156	140
130	3,023	151	172	191	196	195
		3,617	4,231	4,824	4,958	4,956
		193	182	166	150	133
140	3,307	161	182	195	197	195
		3,913	4,530	4,928	5,002	4,939

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 14.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 220 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
		[1]241	253	258	255	246
60	1,185	81	103	126	150	175
		1,700	2,283	2,920	3,604	4,340
		236	244	245	240	229
70	1,428	91	113	135	159	183
		1,957	2,543	3,177	3,868	4,581
		232	236	234	226	213
80	1,679	101	123	145	169	192
		2,225	2,819	3,462	4,148	4,840
		228	229	224	214	199
90	1,937	111	133	155	178	200
		2,495	3,098	3,751	4,423	5,107
		224	222	215	203	186
100	2,200	121	143	166	188	208
		2,780	3,392	4,051	4,31	5,341
		221	216	207	192	174
110	2,469	132	153	176	198	208
		3,069	3,697	4,356	5,023	5,355
		218	211	199	183	164
120	2,744	142	164	186	205	209
		3,356	4,002	4,671	5,262	5,369
		216	206	192	174	156
130	3,023	152	174	196	208	209
		3,656	4,316	4,980	5,330	5,365
		213	202	186	167	148
140	3,307	162	184	205	209	208
		3,954	4,618	5,262	5,387	5,355

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 15.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 240 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
		[1]262	275	279	276	266
60	1,185	82	106	130	156	182
		1,733	2,357	3,033	3,68	4,551
		257	265	266	260	248
70	1,428	92	115	140	165	190
		1,993	2,620	3,295	4,032	4,791
		253	257	255	246	233
80	1,679	102	125	149	174	199
		2,259	2,889	3,571	4,304	5,055
		248	250	245	234	218
90	1,937	112	135	160	184	208
		2,528	3,176	3,873	4,612	5,344
		245	243	235	222	205
100	2,200	122	146	170	194	217
		2,813	3,468	4,175	4,900	5,614
		242	237	227	212	192
110	2,469	133	156	180	203	221
		3,104	3,773	4,478	5,197	5,735
		239	232	219	202	181
120	2,744	143	167	190	213	221
		3,392	4,081	4,797	5,510	5,756
		236	227	212	193	172
130	3,023	153	177	201	219	221
		3,693	4,393	5,110	5,690	5,765
		234	222	205	185	164
140	3,307	163	187	210	221	221
		3,989	4,701	5,414	5,757	5,760

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

APPENDIX II

FORTRAN PROGRAM FOR ELM-ASH-COTTONWOOD YIELD SIMULATION

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C***********************************************************************
C  THIS IS A SIMPLE PROGRAM THAT INTEGRATES THE SYSTEM OF
C  ELM-ASH-COTTONWOOD DIFFERENTIAL EQUATIONS DESCRIBING
C  CHANGE IN NUMBER OF TREES AND BASAL AREA PER ACRE.
C  THIS PROGRAM CALLS THE IBM SCIENTIFIC SUBROUTINE RKGS
C  WHICH DOES THE ACTUAL NUMERICAL INTEGRATION.  FOR MORE
C  INFORMATION ON RKGS REFER TO :
C      INTERNATIONAL BUSINESS MACHINES CORPORATION. 1968.
C      SYSTEM/360 SCIENTIFIC SUBROUTINE PACKAGE
C      (360-CM-03X) VERSION III PROGRAMMER'S MANUAL.
C      4TH ED. TECHNICAL PUBLICATIONS DEPARTMENT, WHITE PLAINS,
C      NEW YORK.  452 P.
C  INPUTS REQUIRED ARE INITIAL NUMBER OF TREES PER ACRE,
C  INITIAL BASAL AREA PER ACRE, LENGTH OF DESIRED PROJECTION,
C  AND FREQUENCY OF PRINTED OUTPUT.  ALL VALUES ARE FOR TREES
C  5 INCHES DBH AND LARGER.
C***********************************************************************
      EXTERNAL FCT,OUTP,RKGS
      DIMENSION PRMT(5),Y(7),DERY(7),AUX(8,7)
      COMMON TNT1,BAT1,TIMER,PERIOD,REPORT
      TIMER = 0.0

C***********************************************************************
C  READ IN INITIAL NUMBER OF TREES, INITIAL BASAL AREA,
C  PROJECTION PERIOD, AND REPORTING INTERVAL.
C      TNT1 = NUMBER OF TREES PER ACRE 5 INCHES DBH OR LARGER
C      BAT1 = BASAL AREA OF THE TNT1 TREES
C      PERIOD = DESIRED PROJECTION LENGTH IN YEARS
C      REPORT = YEARS BETWEEN REPORTED OUTPUT
C***********************************************************************
      READ(5,500)TNT1,BAT1,PERIOD,REPORT
  500 FORMAT(2F4.0,2F3.0)

C***********************************************************************
C  INITIALIZE ARRAY PRMT (USED BY RKGS)
C      PRMT(1) = LOWER BOUND OF INTEGRATION
C      PRMT(2) = UPPER BOUND OF INTEGRATION
C      PRMT(3) = INITIAL STEP SIZE
C      PRMT(4) = ERROR TOLERANCE
C***********************************************************************
      PRMT(1) = 0.0
      PRMT(2) = PERIOD + .1
      PRMT(3) = 0.1
      PRMT(4) = 0.001
```

```
C***************************************************************************
C   INITIALIZE ARRAY Y WITH THE INITIAL CONDITIONS FOR EACH
C   OF THE DIFFERENTIAL EQUATIONS IN THE SIMULATOR.
C        Y(1) = CUMULATIVE NUMBER OF INGROWTH TREES PER ACRE
C        Y(2) = CUMULATIVE NUMBER OF MORTALITY TREES PER ACRE
C        Y(3) = NUMBER OF TREES PER ACRE.
C        Y(4) = CUMULATIVE BASAL AREA INGROWTH PER ACRE
C        Y(5) = CUMULATIVE BASAL AREA SURVIVOR GROWTH PER ACRE
C        Y(6) = CUMULATIVE BASAL AREA MORTALITY PER ACRE
C        Y(7) = BASAL AREA PER ACRE
C   VALUES MUST BE GREATER THAN ZERO.
C***************************************************************************
      Y(1) = 0.00001
      Y(2) = 0.00001
      Y(3) = TNT1
      Y(4) = 0.00001
      Y(5) = 0.00001
      Y(6) = 0.00001
      Y(7) = BAT1

C***************************************************************************
C   INITIALIZE THE ARRAY NDIM TO THE TOTAL NUMBER OF EQUATIONS
C***************************************************************************
      NDIM=7

C***************************************************************************
C   INITIALIZE THE ARRAY OF EQUATION WEIGHTS, DERY.
C***************************************************************************
      DO 10 I=1,NDIM
      DERY(I)=1.0/FLOAT(NDIM)
   10 CONTINUE


C***************************************************************************
C   WRITE OUTPUT TABLE HEADING.  CFV IS CUBIC FOOT VOLUME
C   ESTIMATED AS A FUNCTION OF BASAL AREA.
C***************************************************************************
      CFV=8.319*Y(7)**1.2112
      WRITE(6,600)TNT1,BAT1,CFV
  600 FORMAT(1H1,17X,34HSIMULATED ELM-ASH-COTTONWOOD YIELD,/
     +10X,51H(PER ACRE VALUES FOR TREES 5 IN. D.B.H. AND LARGER),/19X,
     +31H(TREES 5 INCHES DBH AND LARGER)//19X,
     +23HINITIAL NUMBER OF TREES,
     +F6.0/19X,27HINITIAL BASAL AREA (FT.SQ.),F6.0/
     +19X,25HINITIAL CUBIC FOOT VOLUME,F6.0///
     +60H         ELAPSED          NUMBER           BASAL     CUBIC FOOT/
     +60H            TIME        OF TREES            AREA         VOLUME/)

C***************************************************************************
C   CALL THE RKGS SUBROUTINE TO INTEGRATE THE SYSTEM OF
C   DIFFERENTIAL EQUATIONS.
C***************************************************************************
      CALL RKGS(PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
      STOP
      END
```

```
      SUBROUTINE FCT(X,Y,DERY)
      DIMENSION Y(7),DERY(7)
      COMMON TNT1,BAT1,TIMER,PERIOD,REPORT
C************************************************************************
C  THIS SUBROUTINE HOLDS THE RIGHT HAND SIDES OF THE
C  DIFFERENTIAL EQUATIONS TO BE SOLVED SIMULTANEOUSLY.
C  DERY(1) IS THE DERIVATIVE OF Y(1), DERY(2) IS THE
C  DERIVATIVE OF Y(2), ETC.
C************************************************************************
      DERY(1) = 10.139 * EXP(0.0024561 * Y(3) - 0.015602 * Y(7))

      DERY(2) = 1.1061 * EXP(0.0065612 * Y(7))

      DERY(3) = DERY(1) - DERY(2)

      DERY(4) = 0.13635 * DERY(1)

      DERY(5) = 0.030429 * SQRT(Y(7) * Y(3))

      DERY(6) = (((3.1424*((-ALOG(1.0-RANF(0)))**1.1041)) + 5.0)
     2**2)*0.005454*DERY(2)
      QUAD = Y(7)/Y(3)
      IF(QUAD.GT.1.1.OR.QUAD.LT.0.40) DERY(6) = QUAD*DERY(2)

      DERY(7) = DERY(4) + DERY(5) - DERY(6)

      RETURN
      END



      SUBROUTINE OUTP(X,Y,DERY,IHLF,NDIM,PRMT)
      DIMENSION Y(NDIM),DERY(NDIM),PRMT(5)
      COMMON TNT1,BAT1,TIMER,PERIOD,REPORT

C************************************************************************
C  SUBROUTINE OUTP PRINTS SELECTED OUTPUT FROM THE SOLUTION
C  PROCESS.  THIS SUBROUTINE IS CALLED AT LEAST ONCE EVERY
C  PRMT(3) YEARS.  AT EACH CALL TO OUTPUT THE VALUES OF
C  ARRAY Y (THE CURRENT CUMULATIVE YIELD VALUES) AND X
C  (THE TIME VARIABLE) ARE PASSED TO SUBROUTINE OUTP.  THESE
C  VALUES ARE PRINTED EVERY "REPORT" YEARS.
C************************************************************************
      ELAPSED=X-TIMER
      IF(ELAPSED.GE.(REPORT-.001))GO TO 5
      GO TO 10

    5 CFV=8.319*Y(7)**1.2112
      WRITE(6,600)X,Y(3),Y(7),CFV
  600 FORMAT(4(5X,F10.0))
     +,Y(6)
      TIMER = TIMER + REPORT

   10 RETURN
      END
```

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Shifley, Stephen R.; Moser, John W., Jr.; Brown, Kenneth M.
Growth and yield model for the elm-ash-cottonwood type in Indiana. Res. Pap. NC-218. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station: 1982. 16 p.

Presents a growth and yield model for the elm-ash-cottonwood forest type in Indiana and discusses model validity. Gives simulated yield estimates for a wide range of stand conditions.

KEY WORDS: Bottomland hardwoods, yield tables, simulation.

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