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Self-organized criticality in forest-landscape evolution

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Abstract

A simple cellular automaton replicates the fractal pattern of a natural forest landscape and predicts its evolution. Spatial distributions and temporal fluctuations in global quantities show power-law spectra, implying scale-invariance, characteristic of self-organized criticality. The evolution toward the SOC state and the robustness of that state to perturbations are described.
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The idea that natural systems self-organize into a critical state far from equilibrium that exhibits spatial and temporal scale-invariance with power-law spectra is an important paradigm for understanding the behavior of complex systems [1], especially biological systems [2]. Self-organized criticality (SOC) has been observed in many such systems including earthquakes [3], quasar luminosities [4], sand-piles [5], chemical reactions [6], and biological evolution [7], as well as simple computational models of such processes. We present here evidence that a natural forest landscape exhibits similar spatial scale-invariance. We develop a simple two-dimensional cellular automaton (CA) model with a single adjustable parameter that replicates the spatial structure in the long-time limit, independent of the initial condition. The model exhibits realistic temporal fluctuations that are also scale-invariant over a wide range of time scales. More complicated forest models with similar behavior have been previously studied [8,9], especially involving forest fires [10–12].

The U.S. General Land Office Surveys are widely recognized to provide a reliable basis for large-scale quantitative and qualitative information on landscapes prior to major Euro-American influence. Carried out in the 19th century from Ohio to the west coast of the United States, the surveys divide the land into a regular 1-square-mile grid. Along the grid, survey posts were set, and one to four trees near the posts were blazed (witness trees). Information such as the tree species or distance from the corner was recorded, along with additional characteristics of the landscape (e.g., swamps, burns, or windfalls).

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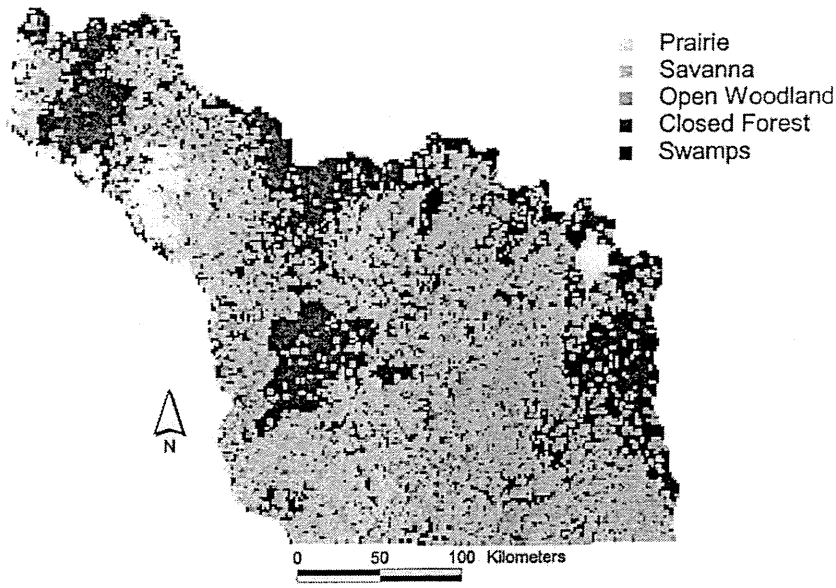

Fig. 1. Landscape of Southern Wisconsin in the early to mid 1800's.

These surveys have been widely used to address a broad range of scientific questions such as reconstructing and restoring historical landscapes [13–17], landscape classification schemes [18–21], investigation of historical landscape-scale disturbances such as wind [22–25] and fire [20,26], analysis of the historic landscape pattern and change [15,23,27–29], and research on the anthropogenic role of land-use change [14] and early socio-economic trends [30].

The U.S. General Land Office Surveys for southern Wisconsin are shown in Fig. 1. The surveys consist of 27,886 measurements of landscape types on a one-mile (1.6 km) grid. For our purpose, each grid point is characterized by a discrete landscape type corresponding to a range of tree densities [13]. These types include prairie (<0.5 trees/ha), savanna (0.5–46 trees/ha), open woodland (46–99 trees/ha), and closed forest (>99 trees/ha) (1 ha $= 1 \times 10^4$ m^2). Additionally, we distinguish swamps, as well as a separate category for missing data. The results are not sensitive to the details of the categorization.

Spatial scale-invariance was tested using a form of the Grassberger–Procaccia algorithm [31]. The correlation sum $C(\varepsilon)$ is defined as the probability that two randomly chosen points (1 and 2) of the same integer value (0–5) are within a distance ε of one another, where ε is taken as the maximum norm $|x_1 - x_2| + |y_1 - y_2|$ rather than the usual Euclidean norm. If the plot of $\log C(\varepsilon)$ versus $\log \varepsilon$ is a straight line, the object is spatially scale-invariant (a fractal), and the slope of the line is the fractal dimension. Fig. 2 shows such a plot for the data in Fig. 1, indicating spatial scale-invariance from the smallest scale (the cell size) up to a scale approaching the size of the image, with a fractal dimension of about 1.6. Similar fractal structure has been observed in tropical rainforests [8,9].

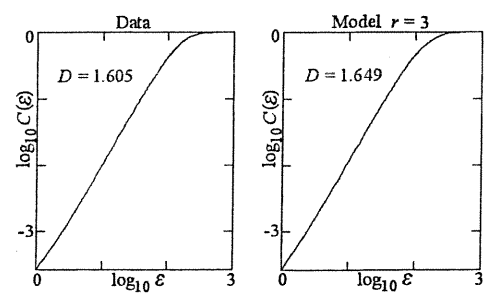

Fig. 2. Correlation sum for landscape data and model showing spatial scale-invariance with a fractal dimension of about 1.6.

In addition to the fractal dimension, another point of comparison between the data and the model to be described is the probability that an arbitrary point in the array is part of a cluster of identical points, here defined as its four nearest neighbors (a von Neumann neighborhood). A highly disorganized array will have

a small cluster probability, and a highly organized array will have a large cluster probability. The cluster probability for the data in Fig. 1 is 32.3%.

The CA model consists of a rectangular array of 253 × 202 cells to match the resolution of the observed data with periodic boundary conditions (a torus), although the results are not sensitive to the boundary conditions or array size. Each cell has one of six values chosen with the same probability as the data. For comparison, only those cells in the array that overlap the region of observation are considered, although all cells in the rectangular array are evolved.

The rule is extremely simple. At each time step, a randomly chosen cell in the array is replaced by a cell chosen randomly from its circular neighborhood of radius r (measured in units of the cell size). The radius r is the only parameter in the model. After 51,106 such steps, each cell in the array has been replaced once on average, and this time is called a generation. It corresponds roughly to the average life of a tree, on the order of 100 years. Note that this method differs from the usual CA in which all cells are updated simultaneously. The difference is of little consequence as evidenced by tests in which the cells chosen for replacement are highly correlated with previous replacements as might occur for disease or fire. The chosen procedure is computationally convenient and models a forest in which trees die somewhat randomly.

For calculations longer than a few thousand generations, there is a tendency for the minority categories to become extinct. To prevent that from happening, a very small feedback is added that occasionally suppresses replacements whenever such replacements would cause the number in that category to exceed the desired value. This embellishment is not critical to the results, but it allows very long simulations, corresponding to millions of years.

The correlation sum for the model with $r = 3$ and random initial conditions is shown in Fig. 2 after 10^4 generations. The plot is nearly a straight line with a fractal dimension of about 1.65, which is similar to the observed data. The fractal dimension is not the best metric for testing the model, however, since it is less sensitive than other measures.

The final state is insensitive to the initial conditions. Two extreme cases are random initial conditions having a very low cluster probability (about 4%) and a highly ordered one with a high cluster probability

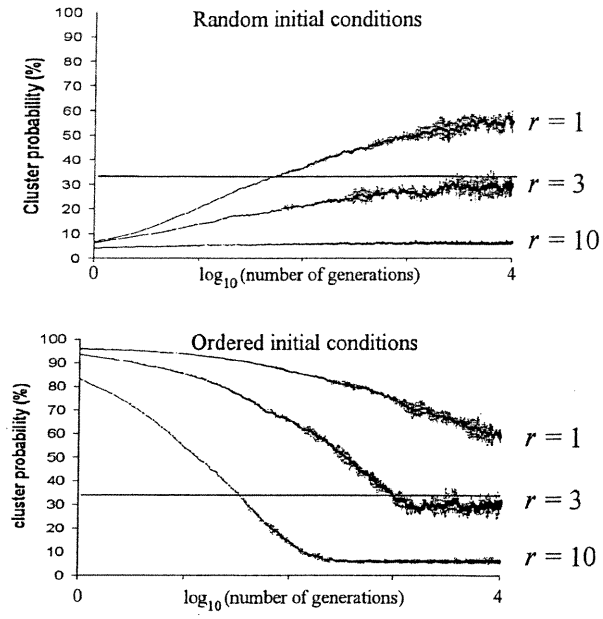

Fig. 3. Evolution of the cluster probability shows that $r = 3$ self-organizes to a value close to the observed data for two extreme initial conditions.

(about 96%). Fig. 3 shows how the cluster probability evolves for these initial conditions for three values of r. Each plot is an average of five runs with different sequences of random numbers, showing ± one standard deviation in gray. The case with $r = 3$ converges to a value close to the observed data after about a thousand generations, independent of initial conditions. Smaller values of r over-organize, and larger values of r under-organize. There does not appear to be a sharp threshold as with a phase transition.

Resampling the observed data at different resolutions gives the same fractal dimension and cluster probability as expected with spatial scale invariance. Although the model best fits the observation for $r = 3$, there is nothing special about a distance of 3 miles (~5 km). Rather, the critical parameter for $r = 3$ appears to be the $9\pi \sim 28$ cells from which replacements are chosen.

After the CA self-organizes, there are temporal fluctuations in global quantities such as the cluster probability. The power spectrum of these fluctuations sampled once per generation was calculated using the maximum entropy method [32], and the results for $r = 3$ and $r = 10$ are shown in Fig. 4. The

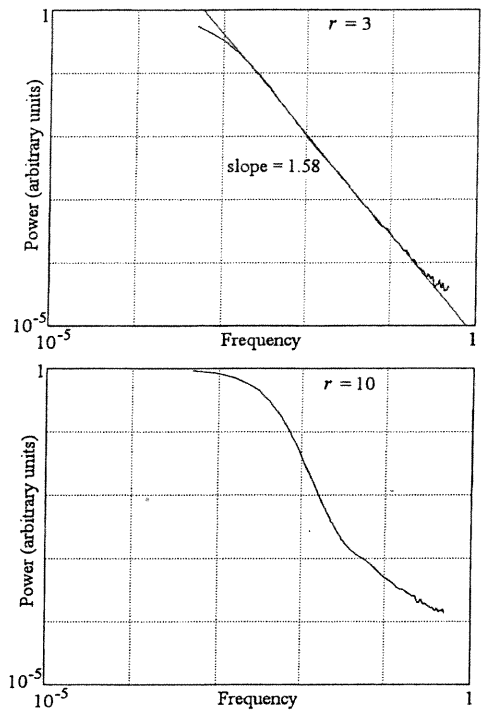

Fig. 4. Power spectra for fluctuations of the cluster probability show temporal scale-invariance at small r with a scaling exponent of about 1.58.

frequency is measured in units of the inverse time for one generation (about 100 years), and the power is in arbitrary units. A frequency of 10^{-3} thus corresponds to about 10^5 years. The case with $r = 1$ (not shown) is indistinguishable from the case with $r = 3$. The self-organized state exhibits a power-law spectrum with a spectral exponent of about 1.58, indicating temporal scale-invariance at least over the range of a few hundred years to about 10^5 years. The case with $r = 10$ does not self-organize and does not have a power-law spectrum, implying that there are characteristic time scales and the resulting state is not SOC.

Another measure of the degree of organization is the algorithmic complexity, which is the size of the smallest computer program that will replicate the pattern [33,34]. A simple way to estimate this quantity uses the fact that GIF graphics files are nearly optimally compressed. Thus one only needs to determine the size of the GIF file of the image after subtracting the fixed size of the file header. The image representing the observed data in Fig. 1 compresses to 6205 bytes. The CA model with random initial conditions has a size of 8136 bytes, which with $r = 3$ evolves after many generations to a size of 6782 bytes, with small temporal fluctuations. Although the significance of such a measure is subject to debate [35], it nonetheless provides a point of comparison between a model and the data.

The robustness of the SOC state was studied by randomly exchanging some cells after a self-organized state was reached to maintain the same probability of categories. The perturbed and unperturbed cases then evolved using the same rules with the same sequence of random numbers, and the number of cells that differ in the two cases was calculated at each generation. Perturbations with sizes up to about half the total number of cells eventually die, and the two cases become identical. The reason is that the site from which a replacement is chosen is more likely to be the same than different in the perturbed and unperturbed cases if less than half the sites are exchanged. The average number of generations required for the perturbation to die completely is the order of the number of sites perturbed. In this sense, the model is stochastic but not chaotic, and the resulting state is strongly resistant to perturbations—an optimistic result from the ecological perspective.

Note also that the model does not invoke any particular biological mechanism for the death of the cells and replacement by neighboring cells. Trees can die from age, disease, fire, or human activity without altering the results. It is only necessary that the replacements come from the near neighborhood. To the extent that the model captures the dynamics, the SOC state is reached by whatever mechanisms nature provides, although the fitted value of $r = 3$ may have some biological significance, yet to be determined. The same model with an appropriate choice of r would probably suffice to explain other similar systems such as the migration of human populations and the spread of disease throughout a plant or animal population.

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References

[1] P. Bak, How Nature Works: The Science of Self-Organized Criticality, Copernicus, New York, 1996.
[2] T. Gisiger, Biol. Rev. 76 (2001) 161.
[3] B. Gutenberg, C.F. Richter, Seismicity of the Earth and Associated Phenomena, Princeton University Press, Princeton, NJ, 1954.
[4] W.H. Press, Comments Astrophys. 7 (1978) 103.
[5] P. Bak, C. Tang, K. Wiesenfeld, Phys. Rev. Lett. 59 (1987) 381.
[6] R.H. Simoyi, A. Wolf, H.L. Swinney, Phys. Rev. Lett. 49 (1982) 245.
[7] P. Bak, K. Sneppen, Phys. Rev. Lett. 24 (1993) 4083.
[8] R.V. Solé, S.C. Manrubia, J. Theor. Biol. 173 (1995) 31.
[9] R.V. Solé, S.C. Manrubia, Phys. Rev. E 51 (1995) 6253.
[10] B. Drossel, F. Schwabl, Phys. Rev. Lett. 69 (1992) 1629.
[11] P. Sinha-Ray, L.B. de Água, H.J. Jensen, Physica D 157 (2001) 186.
[12] B.D. Malamud, G. Morein, D.L. Turcotte, Science 281 (1998) 1840.
[13] R.C. Anderson, M.R. Andersen, Castanea 40 (1975) 345.
[14] E.W.B. Russell, Am. Midland Natural. 105 (1981) 1.
[15] E.W.B. Russell, People and the Land Through Time: Linking Ecology and History, Yale University Press, New Haven, CT, 1997.
[16] J.J. Batek, W.A. Rebertus, T.L. Schoreder, E.C. Haithcoat, R.P. Guyette, J. Biogeography 26 (1999) 397.
[17] V.C. Radeloff, D.J. Mladenoff, M.S. Boyce, Restor. Ecol. 8 (2000) 119.
[18] R.W. Finley, Ph.D. thesis, University of Wisconsin, Madison (1976).
[19] L.A. Leitner, C.P. Dunn, G.R. Guntenspergen, F. Stearns, D.M. Sharpe, Landscape Ecol. 5 (1991) 203.
[20] D.G. Brown, Plant Ecol. 134 (1998) 97.
[21] K.L. Manies, D.J. Mladenoff, Landscape Ecol. 15 (2000) 741.
[22] G.G. Whitney, J.R. Steiger, Botan. Gaz. 146 (1985) 421.
[23] Q. Zhang, K.S. Pregitzer, D.D. Reed, Am. Midland Natural. 143 (1999) 94.
[24] C.D. Canham, O.L. Loucks, Ecology 65 (1984) 803.
[25] L.E. Frelich, C.G. Lorimer, Ecol. Monogr. 61 (1991) 145.
[26] V.M. Kline, G. Cottam, Ecology 60 (1979) 861.
[27] D.J. Mladenoff, E.A. Howell, Trans. Wisconsin Acad. Sci. Arts Lett. 68 (1980) 74.
[28] M.A. White, D.J. Mladenoff, Landscape Ecol. 9 (1994) 191.
[29] V.C. Radeloff, D.J. Mladenoff, H.S. He, M.S. Boyce, Canad. J. Forest Res. 29 (1999) 1649.
[30] J. Silbernagel, S.R. Martin, M.R. Gale, J. Chen, Landscape Ecol. 12 (1997) 223.
[31] P. Grassberger, I. Procaccia, Phys. Rev. Lett. 50 (1983) 346.
[32] W.H. Press, B.P. Flannery, S.A. Teukolsky, W.T. Vetterling, Numerical Recipes: The Art of Scientific Computing, Cambridge University Press, Cambridge, 1993.
[33] A. Lempel, J. Ziv, IEEE Trans. Inform. Theory IT-22 (1976) 75.
[34] F. Kaspar, H.G. Schuster, Phys. Rev. A 36 (1987) 842.
[35] D. Feldman, J.P. Crutchfield, Phys. Lett. A 238 (1998) 244.