Diameter Growth Models Using Minnesota Forest Inventory and Analysis Data

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ABSTRACT. The Forest Inventory and Analysis (FIA) program of the USDA Forest Service North Central Research Station (NCRS) has begun replacing the 12- to 13-yr periodic inventory cycles for the states in the North Central region with annual inventories featuring measurement of approximately 20% of all plots in each of the 11 states each year. State reports on summaries of the forest resources will be produced every 5 yr. As a method of updating information on plots not visited in the current year, NCRS is developing nonlinear, individual-tree, distance-independent annual diameter growth models for species groups. The models, formulated as the product of an average diameter growth component and a modifier component, were calibrated on Minnesota FIA data from stands that were generally undisturbed, of mixed ages and of mixed species. The dependent variable is annual diameter growth. The independent variables include crown ratio, crown class, stand basal area, stand basal area larger than the subject tree, physiographic class, and latitude and longitude of plot locations. The model predictions at both the individual-tree level and plot level have negligible bias, and the models may be easily recalibrated to include new data sets obtained from the annual inventories. For. Sci. 47(30):301–310.

Key Words: Average diameter growth model, gamma probability distribution function, individual-tree, distance-independent.

In response to the 1998 Farm Bill, formally known as the Agricultural Research Extension and Education Reform Act, the North Central Research Station (NCRS) of the USDA Forest Service has developed an annual inventory system featuring a hexagonal grid of Forest Inventory and Analysis (FIA) plots to be measured in 5 yr inventory cycles, with 20% of the plots to be measured each year (Brand et al. 2000). State reports summarizing the 5 yr inventory of forest resources will be published. Because inventories are conducted over 5 yr cycles, data from 80% of the plots will be 1 to 4 yr old, and summaries based on moving averages lag current conditions. Individual tree diameter growth models provide a means to eliminate this lag by estimating current conditions for FIA plots not measured in the current year.

Historically, growth and yield models have been generally classified into three categories (Davis and Johnson 1987): (1) whole stand models that predict growth for the entire stand based on a stand characteristic such as basal area, age, or site index (e.g., Buckman 1962, Clutter 1963, Moser and Hall 1969, Sullivan and Clutter 1972, McRoberts 2000); (2) diameter class models that predict stand growth rates by diameter classes among which the number of trees per unit area are distributed (e.g., Bailey and Dell 1973, Adams and Ek 1974, Ek 1974, Solomon et al. 1995); and (3) individual tree models that predict growth for individual trees generally based on a combination of tree and stand characteristics. The individual tree models that predict diameter growth may be further divided into distance-dependent models that require that the data include mapped tree locations so that competition information may be incorporated (e.g., Daniels and Burkhart 1975, Daniels and Burkhart 1988) and distance-independent models that do not require mapped tree loca-

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tions, but use competition factors such as stand basal area or stand density (e.g., Stage 1973, Belcher et al. 1982, Murphy and Sternitzke 1979, Shifley and Fairweather 1983, Shifley 1987, Amateis et al. 1989, Hasenauer et al. 1998, Murphy and Graney 1998, Cao 2000). The growth models developed by Stage (1973), Murphy and Sternitzke (1979), Belcher et al. (1982), Shifley and Fairweather (1983), and Shifley (1987) have been applied to FIA data for predicting growth. Of these models, FIA data was used to calibrate the models developed by: (1) Murphy and Sternitzke (1979) for loblolly pine (*Pinus taeda* L.) on 145 FIA plots in the West Gulf; (2) Shifley and Fairweather (1983) for all species on 831 FIA plots in western Oregon; and (3) Shifley (1987) for all species groups on more than 2,700 FIA plots in the central states. The latter two sets of models were based on the STEMS (Belcher et al. 1982) model formulation. The diameter growth models developed in this article are individual-tree distance-independent models that are calibrated on FIA data.

The FIA program of NCRS has used STEMS (Belcher et al. 1982) to predict individual tree diameter growth and survival. The current methodology for calibrating models departs from that used for STEMS in order to improve the efficacy of the models in several areas:

- The primary intended application of the models reported in this study is in an inventory context. Therefore, the calibration data have been restricted to inventory data, whereas the source of much of the data used to calibrate the STEMS models was long-term research plots.
- The mathematical form of the current models avoids the potential growth construct common to STEMS and other growth models, because potential growth cannot be observed and historical procedures for estimating it are complex and time-consuming.
- The current models were calibrated using a traditional regression procedure, which facilitates accurate estimation of the uncertainty of model predictions. The two-step procedure used to calibrate the STEMS models first estimated the parameters of the potential component using a subset of the data, fixed these parameter estimates, and then calibrated the modifier component. This procedure precludes accurate estimation of parameter covariance and model prediction uncertainties: (1) no estimates of the covariance between the parameters of the potential and modifier components are possible; (2) bias in the model predictions may occur, because the potential component parameters are not allowed to vary in response to the totality of observed data; and (3) the uncertainty of model predictions cannot be accurately obtained, because the parameter covariance structure is incomplete.

The new models partially address the underlying objectives of negligible bias when applied in an inventory context, easy recalibration with inventory data collected under the annual system, and accurate estimation of the uncertainty of model predictions.

The objectives of this study were to develop individual-tree, distance-independent diameter growth models using FIA data collected in previous inventories. The form of the models will be the product of an average diameter growth component and a modifier component, and will be calibrated as a single unit. The models, calibrated for major species groups within Minnesota, were to have negligible bias and be of a form that can easily be recalibrated with inventory data collected under the annual system.

Data

The diameter growth models were calibrated using Minnesota FIA data obtained across all ownership categories on land classified as timberland. Timberland is defined as nonreserved forestland that is producing or is capable of producing 1.4 m^3/ha/yr of industrial wood. Although the models were calibrated on data from 3,158 plots with no disturbance or small disturbance over the measurement interval, some of the plots had substantial mortality or harvesting prior to the measurement interval. The effect of any resulting bias in the models will be tempered by predictor variables such as plot basal area that partially account for disturbance when predicting growth rates and because the proportion of extensively disturbed plots is low. Data for trees with diameters of 12.7 cm or greater were collected on variable-radius point clusters (BAF 8.61 m^2/ha), while data for trees with diameters between 2.5 and 12.7 cm were collected on clusters of 0.008 ha fixed-radius plots (Hansen et al. 1992). FIA data from the Minnesota 1977 and 1990 inventories were used to calibrate the growth models. Only remeasured trees that were alive during both inventories were used to calibrate the individual-tree diameter growth models.

Both tree-level and plot-level variables were included in the diameter growth models. At the tree level, diameter at breast height (DBH) is used as the major predictor variable in the growth model for predicting change in DBH. The minimum DBH of trees in the data set is 2.5 cm. Annual change in DBH is the dependent variable for application of the diameter growth models. Because annual measurements were not available in the data set, average annual change in DBH (calculated as the ratio of the difference in DBH measurements at the two inventories and the number of years between measurements) is used as a surrogate for annual diameter growth in model calibration. Other individual tree variables include initial crown ratio (CR) and crown class (CC). Crown ratio is the percentage of total tree height that is crown and is assigned in FIA data to one of nine categories. Crown class is recorded in five ordered categories ranging from open-grown to suppressed (Hansen et al. 1992).

Plot-level variables include basal area per hectare (BA), basal area per hectare for trees larger than the subject tree (BAL), physiographic class (PC), latitude (LAT), and longitude (LNG). Information about competition within the stand, given by BAL and BA, are calculated from the plot data. Physiographic class, which is related to site soil and water conditions that affect site productivity, is coded in the data set as 3, 4, 5, 6, or 7 (corresponding with xeric, xeromesic, mesic, hydromesic, and hydric, respectively) and treated as a covariate in the model. Latitude and longitude are surrogates for climatic conditions in the state-wide models.

Table 1. Species included in each modeled species group.

Species group	Species
Softwoods	
Eastern white pine	Eastern white pine (*Pinus strobus* L.)
Red pine	Red pine (*Pinus resinosa* Ait.)
Jack pine	Jack pine (*Pinus banksiana* Lamb.)
White spruce	White spruce (*Picea glauca* (Moench) A. Voss)
Black spruce	Black spruce (*Picea mariana* (Mill.) BSP.)
Balsam fir	Balsam fir (*Abies balsamea* (L.) Mill.)
Tamarack	Tamarack (*Larix laricina* (Du Roi) K. Koch)
Northern white-cedar	Northern white-cedar (*Thuja occidentalis* L.)
Hardwoods	
Select white oaks	Bur oak (*Quercus macrocarpa* Michx.)
	White oak (*Quercus alba* L.)
Northern red oak	Northern red oak (*Quercus rubra* L.)
Hard maple	Sugar maple (*Acer saccharum* Marsh.)
	Black maple (*Acer nigrum* Michx. f.)
Soft maple	Red maple (*Acer rubrum* L.)
	Silver maple (*Acer saccharinum* L.)
Green and white ash	Green ash (*Fraxinus pennsylvanica* Marsh.)
	White ash (*Fraxinus americana* L.)
Black ash	Black ash (*Fraxinus nigra* Marsh.)
Balsam poplar	Balsam poplar (*Populus balsamifera* L.)
Bigtooth aspen	Bigtooth aspen (*Populus grandidentata* Michx.)
Quaking aspen	Quaking aspen (*Populus tremuloides* Michx.)
American basswood	American basswood (*Tilia americana* L.)
Elm	American elm (*Ulmus americana* L.)
	Slippery elm (*Ulmus rubra* Muhl.)
	Rock elm (*Ulmus thomasii* Sarg.)
Paper birch	Paper birch (*Betula papyrifera* Marsh.)
Other commercial	Boxelder (*Acer negundo* L.)
	Yellow birch (*Betula alleghaniensis* Britt.)
	Northern pin oak (*Quercus ellipsoidalis* E. J. Hill)
	Eastern cottonwood (*Populus deltoides* Marsh.)
	Black cherry (*Prunus serotina* Ehrh.)
	Shagbark hickory (*Carya ovata* (Mill.) K. Koch)
	Hackberry (*Celtis occidentalis* L.)
	Butternut (*Juglans cinerea* L.)
	Black walnut (*Juglans nigra* L.)
	Black oak (*Quercus velutina* L.)
	Bitternut hickory (*Carya cordiformis* (Wang.) K. Koch)
	Black willow (*Salix nigra* Marsh.)
	Black locust (*Robinia psuedoacacia* L.)
Noncommercial	Eastern hop-hornbeam (*Ostrya virginiana* (Mill.) K. Koch)
	Mountain maple (*Acer spicatum* Lam.)
	American hornbeam (*Carpinus caroliniana* Walt.)
	Choke cherry (*Prunus virginiana* L.)
	Pin cherry (*Prunus pensylvanica* L. f.)
	Hawthorn (*Crataegus* spp.)

Trees included in the calculation of total plot BA and BAL for time 1 were restricted to all living trees recorded on the plot in the first inventory. Trees included in the calculation of total plot BA and BAL at time 2 were restricted to the remeasured trees that were alive and included in the plot at time 1 and still alive at time 2. Plot variables associated with individual trees were maintained with each tree record. The data were split into two data sets to be used in calibrating and validating the models. Every fourth plot was systematically assigned to the validation database. The remaining 75% of the plots were used for calibration of the models. Tree records were sorted into species groups (Table 1). A summary by species group of the combined calibration and validation data is presented in Table 2.

Modeling Methodology

Mathematical Form of the Diameter Growth Model

The form of the diameter growth models is the product of two components, average DBH growth and a modifier. The average component gives the average growth rates by initial diameter for a species group across all plot and tree conditions for the state of Minnesota. The average diameter growth component replaces the potential diameter growth component common to some growth models. The average component is based on a two-parameter gamma function utilizing DBH as the independent variable to predict annual diameter growth rates. The modifier is a product of exponentials, of which each incorporates a single additional independent variable. The modifier gives greater prediction capability to

Table 2. Summary of the 1977–1990 Minnesota FIA data used to form the calibration and validation data sets and to calibrate the final diameter growth models.

Species group	No. of trees	Dbh Median	Dbh Minimum	Dbh Maximum	Species	Relative group total (%)
		.... (cm)	....			
Softwoods						
Eastern white pine	411	39.6	2.8	99.8	Eastern white pine	100
Red pine	890	32.5	3.3	66.5	Red pine	100
Jack pine	1,447	22.4	2.8	63.8	Jack pine	100
White spruce	482	25.7	3.6	71.4	White spruce	100
Black spruce	4,341	11.9	2.5	52.3	Black spruce	100
Balsam fir	3,432	14.5	2.5	51.6	Balsam fir	100
Tamarack	1,876	15.5	2.5	46.7	Tamarack	100
Northern white-cedar	3,183	21.1	2.5	71.6	Northern white-cedar	100
Hardwoods						
Select white oaks	1,372	25.4	2.8	91.9	Bur oak	94
					White oak	6
Select red oak	1,451	28.2	3.8	93.7	Northern red oak	100
Hard maple	1,547	20.6	2.5	114.8	Sugar maple	100
					Black maple	> 1
Soft maple	1,348	16.8	2.8	126.0	Red maple	92
					Silver maple	8
Green and white ash	320	23.4	2.8	78.7	Green ash	96
					White ash	4
Black ash	2,408	17.3	2.8	58.7	Black ash	100
Balsam poplar	1,862	22.4	3.0	61.5	Balsam poplar	100
Bigtooth aspen	667	27.2	3.3	65.3	Bigtooth aspen	100
Quaking aspen	9,931	23.4	2.8	70.4	Quaking aspen	100
American basswood	1,439	24.4	2.8	79.5	American basswood	100
Elm	417	19.6	2.8	79.0	American elm	91
					Slippery elm	7
					Rock elm	3
Paper birch	4,502	20.1	2.8	64.0	Paper birch	100
Other commercial	552	29.0	3.0	151.6	Boxelder	22
					Yellow birch	22
					Northern pin oak	9
					Eastern cottonwood	9
					Black cherry	8
					Shagbark hickory	6
					Hackberry	4
					Butternut	5
					Black walnut	4
					Black oak	4
					Bitternut hickory	3
					Black willow	3
					Black locust	> 1
Noncommercial	285	5.6	2.5	52.6	E. hop-hornbeam	76
					Mountain maple	10
					American hornbeam	6
					Choke cherry	4
					Pin cherry	2
					Hawthorn	> 1

the growth model by adjusting the predicted growth values for individual tree and stand conditions. The form of the diameter growth model is:

$$E(\Delta DBH) = AVERAGE * MODIFIER, \quad (1a)$$

where

$$AVERAGE = \beta_1 \exp(-\beta_2 DBH) DBH^{\beta_3}, \quad (1b)$$

and

$$MODIFIER = \exp[\beta_4 (CR - 4) + \beta_5 (BAL - 11.5) + \beta_6 (BA - 23) + \beta_7 (CC - 3) + \beta_8 (PC - 5) + \beta_9 (PC - 5)^2 + \beta_{10} (LNG + 93) + \beta_{11} (LAT - 47)]. \quad (1c)$$

The functions within the modifier incorporate aver values of the variables across the state so that under aver conditions the effect of the variable disappears. The gre the deviation of the observed variable's value from average value, the greater the impact that variable has on model prediction.

The parameterization of the gamma function used in average component (1b) of the model is a simplified forr that given by Johnson and Kotz (1970, p. 166). In formulation, β_2 is a scale parameter related to the sprea the distribution, and the shape parameter, β_3, is related to peakedness of the curve. The parameter, β_1, is a multipli better adjust the model fit to the data either upwar downward in conjunction with the rates of annual gro

The parameter estimation routine failed to converge for some species when both β_2 and β_3 were included in the model. Eliminating β_2 or β_3 from the models for those species, thus changing the gamma function to a power function or an exponential function, respectively, allowed the parameter estimation routine to converge for those species groups. Of the two functions, the power function generally provided the better fit, based on the mean square error (MSE). While both the power function and exponential function predict monotonically increasing growth with increasing DBH, this should not present a problem when upgrading 5 yr inventory data.

In addition to the independent variables used in the diameter growth models (DBH, CR, CC, BA, BAL, PC, LAT, and LNG), a number of other independent variables were investigated by Holdaway (2000) for this purpose. These included the number of trees per hectare, average stand diameter, and the ratio of DBH to average stand diameter. Our goal was to build diameter growth models that would explain the most variability while using the smallest number of variables (and associated parameters) to achieve a parsimonious model. No variable was included in the model if the asymptotic 95% confidence interval for the parameter estimate associated with the variable included zero. In addition, a model comparison criterion accounting for both model residual error and the number of estimated parameters was used to determine the variables for inclusion in the models (Linhart and Zucchini 1986, p. 108):

$$C = SSE + 2p * MSE \ , \qquad (2)$$

where C is the criterion, p is the number of estimated model parameters, and SSE and MSE are the model residual sum of squares and mean square error, respectively. Models with lower criterion values are judged to provide a better fit to the data.

Weighted Regression

The models were fit to the average annual growth data, and their growth predictions were compared with the average annual growth observations. Scatterplots of the residuals (observed – predicted) versus the predicted average annual diameter growth showed that variance increased as predicted growth rates increased. Heterogeneity of variance may be addressed by an appropriate weighting scheme (Draper and Smith 1966, p. 147). The residuals were ordered by size and separated into groups, each with an equal number of members. The standard deviation of the residuals and the mean predicted growth value were calculated for each group. Linear regression was used to provide parameter estimates of the model relating standard deviation of the residuals to predicted growth (McRoberts et al. 2000) as:

$$E[\ln(\hat{\sigma})] = \alpha_1 + \alpha_2 \ln(\Delta D\hat{B}H), \qquad (3)$$

where $E(.)$ represents the statistical expectation, $\Delta D\hat{B}H$ is predicted annual diameter growth, $\hat{\sigma}$ is the standard deviation of the residuals for each group described above, and the α's are parameters to be estimated. Equation (3) provided the weights for refitting the nonlinear diameter growth models.

Equation (3) and iteratively reweighted least squares were used to fit each diameter growth model as follows: (1) an unweighted diameter growth model was first fit to the data for a single species; (2) regression analysis as in Equation (3) was conducted on the resulting residuals; (3) the squared inverse of the exponentiated right-hand side of Equation (3) was used to weight observations in the nonlinear regression for the next step in the iteratively reweighted regression process; (4) steps 2 and 3 were repeated to iteratively recalculate weights. For most species, only four reweighting iterations were needed to stabilize the parameter estimates to four decimal places.

Bias Assessments

To verify the fit of the models, the diameter growth models were applied to the calibration data set. Average annual observed change in *DBH* was calculated as the ratio of the difference in *DBH* at the two measurements and the number of years in the measurement interval. The residual was calculated for each tree as the difference of the average annual observed change and predicted annual change in diameter. The data for trees from the 25% of all plots initially set aside were used to validate the models by again applying the models and calculating the residuals. Because the sets of residuals contained a number of extreme values that influenced the values of the common parametric statistics, nonparametric statistics in the form of the 25th percentile, median, and 75th percentile for the distribution of residuals were calculated to examine the prediction bias of the models for both the calibration and validation data. The goodness-of-fit statistic recommended by Kvålseth (1985) for its robustness to the influence of extreme values was calculated for each species group in the data sets as:

$$\textit{Fit index} \quad = \quad 1 - \left[\frac{Med\left(\left|\Delta DBH_i - \Delta D\hat{B}H_i\right|\right)}{Med\left(\left|\Delta DBH_i - \overline{\Delta DBH}\right|\right)} \right]^2 , \qquad (4)$$

where Med(.) is the median absolute difference value and ΔDBH_i, $\Delta D\hat{B}H_i$, and $\overline{\Delta DBH}$ are the observed, the predicted, and the mean annual diameter growth values, respectively. The fit index is analogous to an R^2 statistic, and it provides a measure of the proportion of the total variability explained by the fitted model. The median differences in the numerator and denominator provide resistance to sensitivity caused by extreme outliers. As with traditional R^2, values closer to unity indicate a better fit of the model.

The fit of the models was also assessed on a plot-level basis. Although the models were calibrated on data from plots with little or no disturbance, some mortality or cutting did occur on most of these plots. The models are designed to predict growth and not harvest or mortality. Using plots with cutting or mortality would not give an accurate assessment of the models' abilty to predict plot basal area growth because the removal of trees may cause the observed plot basal area growth to be negative. Of the 2,322 plots in the calibration data set and 836 plots in the validation set, only 262 plots (11.3%) in the calibration set and 88 plots (10.5%) in the validation set had no cutting or mortality during the remeasurement interval. These were used to evaluate the diameter growth models at the plot level. Diameter growth models were applied to tree data on the plots with no cutting or mortality, and predicted values of annual basal area per hectare growth were calculated for each plot. Observed basal area change, calculated as the difference in plot basal area for each plot over the measurement interval, was annualized by dividing by the number of years in the interval. Basal area per hectare growth rate residuals was calculated as the differences in observed and predicted annual change in plot basal area per hectare values. To examine trends in bias of the predictions, the plot-level data were separated by forest type and by categories of initial stand basal area per hectare and number of trees per hectare.

Results

Model Verification and Validation

The results of the analysis of the tree-level residuals for the calibration and validation data sets are given in Tables 3 and 4, respectively. Both parametric and nonparametric statistics are reported as residual means, standard deviations, medians, and interquartile ranges. In forestry journals, residual analysis for growth models is nearly always reported with parametric statistics (e.g., Murphy and Sternitzke 1979, Shifley and Fairweather 1983, Shifley 1987, Amateis et al. 1989, Kowalski and Gertner 1989, Murphy and Graney 1998, Cao 2000, Canavan and Ramm 2000). The mean residuals for the calibration data are all nearly zero (Table 3). The mean residuals for the validation data are generally less than 0.02 cm/yr in magnitude, but with both positive and negative signs, indicating no consistent bias in the models (Table 4). However, the nonparametric statistics provide a better measure of model fit because histograms of the residuals showed slight skewness. The result of this skewness is that nearly all median residuals in both the calibration and validation data are negative, indicating a slight over-prediction by the models. The bias in the models is caused by the distribution of the data on which the models are calibrated. Scatter plots of the average annual diameter growth versus diameter for species groups generally display a small percentage of trees from various diameter sizes that grew at relatively higher rates than the others. The data for these trees pull the predicted curve of the growth model upward. In an attempt to use as much data as possible, we discarded only a small number of outliers in some of the species groups and used iteratively reweighted least squares regression to fit the models.

Although the negative values of the median residuals suggest the models slightly overpredict annual diameter growth, the magnitude of the overprediction is small, ranging from 0.005 to 0.033 cm/yr. Considering that the median diameter growth rate is approximately 0.25 cm/yr in Minnesota and that FIA field crews measure DBH to the nearest 0.25 cm (0.1 in), the size of the residuals indicates that any bias in the models may be considered negligible for their intended application. For the validation set, the magnitude of most median residuals is less than 0.03 cm/yr. Species groups with median residuals larger than 0.03 cm/yr include northern white-cedar, elm, and other commercial hardwoods.

For the calibration set, the fit index values (analogous to R^2, with a values closer to 1.0 indicating a better fit) were generally above 0.300 for both the calibration and validation

Table 3. Residual analysis of the diameter growth models fit to the Minnesota FIA calibration tree-level data. Residuals are calculated as the observed minus predicted diameter growth.

		Parametric statistics		Nonparametric statistics			
Species group	Obs	Mean residual	Standard deviation	Median residual	Inter-quartile range	Median observed diameter growth rate	Fit index
		(cm/yr)					
Softwoods							
Eastern white pine	329	0.000	0.2343	–0.021	0.3310	0.363	0.286
Red pine	652	0.000	0.1538	–0.006	0.1968	0.339	0.438
Jack pine	1,102	0.000	0.1247	–0.006	0.1724	0.233	0.325
White spruce	340	0.000	0.1753	–0.026	0.2123	0.292	0.293
Black spruce	3,346	0.000	0.0865	–0.011	0.0898	0.098	0.453
Balsam fir	2,546	–0.001	0.1337	–0.017	0.1440	0.203	0.582
Tamarack	1,372	0.000	0.1314	–0.026	0.1376	0.127	0.432
N. white-cedar	2,283	0.000	0.0978	–0.011	0.1194	0.148	0.360
Hardwoods							
Select white oaks	1,031	0.000	0.1235	–0.011	0.1469	0.254	0.505
Northern red oak	1,139	0.000	0.1424	–0.019	0.1807	0.318	0.227
Hard maple	1,142	0.000	0.1288	–0.014	0.1549	0.181	0.445
Soft maple	1,009	0.000	0.1581	–0.016	0.1745	0.234	0.363
White and green ash	255	–0.001	0.1642	–0.014	0.1749	0.254	0.585
Black ash	1,764	0.000	0.1019	–0.017	0.1184	0.152	0.406
Balsam poplar	1,408	0.000	0.1318	–0.020	0.1566	0.231	0.182
Bigtooth aspen	520	0.000	0.1517	–0.019	0.1872	0.339	0.120
Quaking aspen	7,207	0.000	0.1623	–0.018	0.2047	0.360	0.246
American basswood	1,096	0.001	0.1581	–0.022	0.2107	0.233	0.195
Elm	304	0.000	0.1871	–0.030	0.2013	0.254	0.394
Paper birch	3,499	0.000	0.1067	–0.012	0.1324	0.162	0.222
Other commercial	442	0.000	0.2089	–0.034	0.2698	0.296	0.282
Noncommercial	207	–0.001	0.0778	–0.019	0.1123	0.106	0.114

Table 4. Residual analysis of the diameter growth models fit to the Minnesota FIA validation tree-level data. Residuals are calculated as the observed minus predicted diameter growth.

		Parametric statistics		Nonparametric statistics			
Species group	Obs	Mean residual	Standard deviation	Median residual	Inter-quartile range	Median observed diameter growth rate	Fit index
		(cm/yr)					
Softwoods							
Eastern white pine	77	0.020	0.2325	0.001	0.3444	0.423	0.306
Red pine	236	0.005	0.1701	–0.024	0.2078	0.394	0.448
Jack pine	345	–0.008	0.1261	–0.005	0.1578	0.254	0.211
White spruce	140	0.032	0.2059	0.000	0.2714	0.327	0.330
Black spruce	985	0.006	0.0873	–0.005	0.1057	0.106	0.337
Balsam fir	885	0.009	0.1391	–0.006	0.1678	0.218	0.467
Tamarack	503	0.007	0.1187	–0.016	0.1435	0.139	0.555
N. white-cedar	898	–0.019	0.0982	–0.036	0.1147	0.117	0.159
Hardwoods							
Select white oaks	341	–0.021	0.1434	–0.033	0.1743	0.234	0.184
Northern red oak	311	0.042	0.1557	0.014	0.2040	0.339	0.370
Hard maple	401	0.007	0.1401	–0.011	0.1563	0.191	0.365
Soft maple	338	0.027	0.1771	–0.002	0.2088	0.254	0.513
White and green ash	65	0.017	0.1428	0.030	0.1539	0.290	0.205
Black ash	644	–0.001	0.1042	–0.014	0.1161	0.139	0.452
Balsam poplar	453	–0.008	0.1346	–0.031	0.1676	0.234	0.082
Bigtooth aspen	146	0.002	0.1626	–0.004	0.1805	0.367	0.385
Quaking aspen	2,720	0.007	0.1653	–0.010	0.2100	0.356	0.252
American basswood	343	0.014	0.1775	0.000	0.2329	0.254	0.253
Elm	110	0.002	0.2002	–0.056	0.2337	0.201	0.196
Paper birch	998	–0.004	0.1158	–0.017	0.1450	0.163	0.112
Other commercial	99	–0.001	0.2425	–0.044	0.2554	0.191	0.392
Noncommercial	76	0.019	0.0993	0.008	0.1097	0.117	0.307

data. Shifley (1987) and Cao (2000) used a parametric version of the fit index, calculated as (Kvålseth 1985):

$$1-\sum\left(\Delta DBH_i-\Delta D\hat{B}H_i\right)^2 / \sum\left(\Delta DBH_i-\overline{\Delta DBH}\right)^2,$$

to evaluate the fit of diameter growth models. Shifley fit diameter growth models for Central States species groups using FIA data, and reported fit index values that ranged from –0.02 to 0.22 for the calibration data and from –0.26 to 0.25 for the validation data. Cao fit diameter growth models to data collected from loblolly pines (*Pinus taeda* L.) in the Southwide Seed Source Study using averaging and iterative modeling techniques, and reported fit index values of 0.19 and 0.21, respectively. Although the parametric form of the fit index is slightly different from the nonparametric form we used in Equation (4), the fit index values reported in Tables 3 and 4 are reasonable for the data.

At the plot level, observed and predicted annual change in basal area per hectare values were first sorted by forest type for both the calibration and validation sets. The magnitude of median residuals for estimates of annual change in basal area per hectare was generally less than 0.1 m^2/ha/yr for all forest types in both the calibration and validation data. Fit index values were generally above 0.600 for both the calibration and validation sets, although the models fit poorly to the 21 black spruce plots in the validation set (Table 5).

When the plot data from the combined forest types were sorted into 5 equal-sized groups by initial plot basal area per hectare, the magnitude of median basal area per hectare growth residuals was within 0.07 m^2/ha/yr for both the calibration and validation data. The fit index values were generally greater than 0.600 for both the calibration and validation sets, but had low values for group 5 of the calibration set and group 4 of the validation set (Table 6). The magnitude of median basal area per hectare growth residuals was also within 0.07 m^2/ha/yr for the data in groups sorted by the initial number of trees per hectare. In the calibration set, the fit index values were generally above 0.600 for both the calibration and validation sets, with a low value for group 5 of the validation set (Table 7).

Parameter Estimates

Because the applications of the models to the calibration and validation data produced only negligibly biased predictions, the data from the two sets were combined, and the models were recalibrated on the entire set of data. Parameter estimates for the Minnesota diameter growth models are given in Table 8. A summary of the combined data used to calibrate the species group diameter growth models is given in Table 2.

Not all variables significantly improve the quality of fit for all species group models. Variables whose associated parameter estimates were not statistically significantly different from zero are indicated in Table 8 by parameter estimates of 0. Species groups with fewer than about 500 to 600 trees are generally more difficult to model and may not include some of the variables that a larger sample would possibly have included. Following DBH, essential to the base model, CR is generally the most important predictor of growth for the species groups. Positive parameters for the crown ratio effect imply that diameter growth rates of trees generally increase as the proportion of the tree height that is crown increases.

Table 5. Residual analysis of estimated plot basal area per hectare growth rates. Residuals are calculated as the observed minus predicted plot basal area per hectare growth.

Data set / Forest type	No. of plots	Residual statistics for annual plot basal area/ha growth: Median	25th percentile	75th percentile	Median observed diameter growth rate	Fit index
		(m²/ha/yr)				
Calibration data						
Jack pine	6	0.00	−0.08	0.02	0.32	0.974
Red pine	7	0.04	−0.25	0.32	0.57	0.472
Balsam fir	8	0.02	−0.01	0.13	0.33	0.955
Black spruce	63	0.01	−0.07	0.09	0.22	0.745
N. white-cedar	8	0.07	−0.09	0.14	0.45	0.719
Tamarack	60	−0.01	−0.09	0.06	0.18	0.477
White spruce	2	−0.02	−0.08	0.04	0.42	0.681
Oak-hickory	4	−0.03	−0.13	0.23	0.28	0.778
Elm-ash-cottonwood	20	0.01	−0.04	0.05	0.23	0.942
Maple-beech-birch	6	−0.01	−0.09	0.03	0.39	0.928
Aspen	59	0.03	−0.02	0.10	0.33	0.775
Paper birch	9	−0.02	−0.07	0.04	0.45	0.869
Balsam poplar	8	0.01	−0.04	0.10	0.26	0.976
Validation data						
Jack pine	2	−0.17	−0.20	−0.14	1.15	0.243
Balsam fir	6	0.00	−0.04	0.00	0.02	0.992
Black spruce	21	0.04	−0.08	0.11	0.31	−0.408
N. white-cedar	9	0.01	−0.09	0.11	0.51	0.239
Tamarack	14	−0.03	−0.06	0.00	0.22	0.864
Oak-hickory	6	−0.05	−0.12	0.00	0.32	0.837
Elm-ash-cottonwood	3	−0.18	−0.19	−0.01	0.37	0.692
Aspen	20	−0.01	−0.04	0.05	0.28	0.919
Paper birch	3	0.07	−0.08	0.07	0.35	0.550
Balsam poplar	4	0.08	0.00	0.23	0.18	0.782

Either BAL or BA is the next most important variable for predicting growth. If included in the models, both variables have negative parameters. This indicates a decrease in the growth rate as the stand becomes more crowded. For the shade intolerant species, BAL generally contributes to the quality of model fit; for the shade tolerant species, BA parameter estimates are generally significant in models. The growth of shade intolerant species is detrimentally affected by increases in basal area of trees on the plot with larger diameters than their own. As BAL increases, the subject trees may experience more shaded conditions. The shade tolerant species are able to grow in the understory, but as the overall competition for resources expressed in the form of BA increases, the growth rates decrease.

In even-aged stands, crown class is a measure of a tree's vigor relative to the other trees in the stand and gives a competitive edge to larger trees seeking sunlight in the overstory. A competitive position in the overstory is also important for trees in the uneven-aged stands.

Physiographic class, if included in a model, generally entered as a linear function for species that tolerate wetter soil conditions and quadratically for species that are not found on wet sites. The negative parameter estimates for the linear form of the physiographic class function suggest that as soil conditions become increasingly wetter, the growth rates

Table 6. Residual analysis of estimated plot basal area per hectare growth rates. Plot data were sorted into five groups, each with approximately the same number of observations, by initial plot basal area per hectare. Residuals are calculated as the observed minus predicted plot basal area per hectare growth.

Data set / Group	No. of plots	Initial plot basal area/ha: Median	Minimum	Maximum	Residual statistics for annual plot basal area/ha growth: Median	25th percentile	75th percentile	Median observed diameter growth rate	Fit index
		(m²/ha)			(m²/ha/yr)				
Calibration									
1	52	1.36	0.13	2.22	0.01	−0.01	0.08	0.10	0.894
2	52	3.44	2.34	4.67	0.04	0.00	0.09	0.19	0.840
3	53	6.03	4.68	7.48	0.00	−0.08	0.08	0.24	0.792
4	52	9.63	7.56	12.92	−0.01	−0.07	0.07	0.34	0.604
5	53	16.72	12.94	29.45	−0.06	−0.18	0.12	0.53	0.154
Validation									
1	17	1.64	0.15	2.58	0.00	−0.01	0.04	0.06	0.965
2	18	3.55	2.58	4.50	0.02	−0.01	0.16	0.23	0.754
3	17	6.89	4.50	7.68	−0.07	−0.10	0.05	0.26	0.291
4	18	9.85	7.78	13.59	−0.05	−0.12	0.11	0.34	−2.026
5	18	16.43	13.68	31.45	−0.06	−0.17	0.01	0.46	0.671

Table 7. **Residual analysis of estimated plot basal area per hectare growth rates. Plot data were sorted into five groups, each with approximately the same number of observations, by initial number of trees per hectare. Residuals are calculated as the observed minus predicted plot basal area per hectare growth.**

Data set Group	No. of plots	Initial number of trees/ha			Residual statistics for annual plot basal area/ha growth			Median observed diameter growth rate	Fit index
		Median	Minimum	Maximum	Median	25th percentile	75th percentile		
		(trees/ha)			(m²/ha/yr)				
Calibration									
1	52	121	10	274	0.02	–0.01	0.05	0.10	0.892
2	52	339	277	465	0.03	–0.01	0.08	0.19	0.540
3	53	625	479	823	0.00	–0.07	0.08	0.26	0.695
4	52	1,268	855	1,730	–0.01	–0.12	0.17	0.32	0.611
5	53	2,565	1,732	5,434	–0.01	–0.12	0.14	0.51	0.670
Validation									
1	17	57	13	161	0.00	–0.01	0.01	0.06	0.965
2	18	287	200	541	–0.02	–0.07	0.05	0.25	0.863
3	17	828	544	932	–0.04	–0.12	0.03	0.31	0.631
4	18	1,408	986	1,856	0.03	–0.06	0.14	0.35	0.599
5	18	2,469	1,858	8,587	–0.07	–0.17	0.12	0.42	0.108

decrease for these species even though they tolerate wet sites. The negative parameter estimates of the quadratic form of the physiographic function indicate that there is an optimum soil moisture regime in which the nonwet site species grow best. Deviations toward either drier or wetter conditions decrease growth rates.

Most of Minnesota's timberland is located in the northeastern portion of the state and is contained roughly within a concave line that begins just west of Minnesota's Lake of the Woods region in the north and runs south and east to Minneapolis/St. Paul. Latitude is an important predictor of growth for many of the hardwood species growing close to the northern edge of their range. Negative parameter estimates for latitude indicate that growth rates increase for more southerly plots. Negative parameter estimates for longitude indicate that growth rates increase for more westerly plots. This may be related to the soil structure in the region. The northeastern Arrowhead region of Minnesota, which extends north of Lake Superior, is characterized by rock outcrops and thin soil, while the northwestern part of Minnesota is covered by a layer of glacial till, meltwater deposits, and lake clays which is up to 150 m deep (Ojakangas and Matsch 1982).

Table 8. **Parameter estimates for the annual diameter growth models calibrated on Minnesota FIA data.**[1]

Species group	Average $\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	Modifier $\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$	$\hat{\beta}_{11}$
Softwoods											
E. white pine	0.3673	0	0	0.1214	0	–0.0100	–0.1515	0	–0.2785	0	0
Red pine	0.0708	0.0378	0.7622	0.1347	–0.0257	0	–0.0957	0	0	–0.0500	0
Jack pine	0.1535	0.0328	0.3897	0.1390	–0.0205	0	–0.1072	0	–0.1075	0	–0.1167
White spruce	0.0540	0.0225	0.6411	0.1294	0	–0.0179	0	0	0	0	0
Black spruce	0.0528	0.0256	0.4847	0.1995	0	0	0	–0.1812	0	0	0
Balsam fir	0.0370	0.0400	0.8595	0.1478	0	–0.0113	0	–0.1187	0	–0.0434	–0.0441
Tamarack	0.0421	0	0.5247	0.1745	0	0	0	–0.1950	0	–0.0976	0
N. white-cedar	0.0700	0	0.3128	0.1347	0	–0.0052	0	–0.1115	0	0	–0.1369
Hardwoods	0.0000	0			0	0					
Select white oaks	0.0613	0.0049	0.4244	0.0706	0	–0.0170	–0.1186	0	0	–0.0592	–0.0428
N. red oak	0.1897	0	0.1339	0.0362	–0.0200	0	–0.1315	0	0	0	–0.0492
Hard maple	0.0675	0	0.3777	0.1030	0	–0.0065	–0.0763	0	0	0	0
Soft maple	0.0626	0.0089	0.5806	0.0759	0	–0.0052	0	0	–0.0577	0	–0.1840
White and green ash	0.0889	0	0.3479	0.0788	0	–0.0118	–0.1202	0	0	0	–0.1065
Black ash	0.0745	0	0.3086	0.1275	0	–0.0091	–0.0851	0	–0.0398	–0.1521	–0.1955
Balsam poplar	0.1637	0.0153	0.2541	0.1079	–0.0170	0	0	0	0	0	–0.0611
Bigtooth aspen	0.1704	0	0.2140	0.0979	–0.0166	0	0	0	0	–0.0897	0
Quaking aspen	0.1635	0.0071	0.3163	0.0986	–0.0131	0	–0.0698	0	0	–0.0424	–0.0893
American basswood	0.0966	0	0.3088	0.0878	0	–0.0074	0	0	0	0	–0.0698
Elm	0.0520	0.0144	0.5880	0.1117	0	–0.0113	0	0	0	–0.1013	–0.2413
Paper birch	0.1176	0.0078	0.1969	0.1251	–0.0192	0	–0.0724	0	–0.0486	–0.0851	–0.1340
Commercial	0.1305	0	0.1560	0.0566	0	0	–0.1694	0	0	–0.1675	–0.2090
Noncommercial	0.1293	0	0	0.0857	–0.0218	0	0	0	0	–0.0809	0

[1] The general model is:

$$\Delta DBH = AVERAGE * MODIFIER'$$

where $AVERAGE = \beta_1 \exp(-\beta_2 DBH) DBH^{\beta_3}$, and

$$MODIFIER = \exp[\beta_4 (CR - 4) + \beta_5 (BAL - 11.5) + \beta_6 (BA - 23) + \beta_7 (CC - 3) + \beta_8 (PC - 5) + \beta_9 (PC - 5)^2 + \beta_{10} (LNG + 93) + \beta_{11} (LAT - 47)].$$

Conclusions

The diameter growth models were constructed for species groups using FIA data from Minnesota. Models calibrated using the form and methodology presented here will be used by NCRS for updating information on plots collected under the annual 20% inventory system in the 11 state North Central region. In the context of their intended applications, updating 1 to 4-yr-old information on plots, the bias in the diameter growth models may be considered negligible. The diameter growth models for individual trees generally overpredicted growth by less than 0.025 cm/yr. The bias is small when one considers that FIA field crews measure DBH to the nearest 0.25 cm (0.1 in). The plot-level predictions of basal area per hectare may also be considered to have minimal bias. The median observed average annual basal area per hectare growth for the plots with no cutting or mortality was 1 m^2/ha/yr, the magnitude of the basal area residuals was generally less than 0.1 m^2/ha/yr when examined by forest type or by initial conditions of basal area or number of trees per hectare.

FIA data from Minnesota has been used as a pilot study for broader scale development of diameter growth models calibrated for ecoregions defined by Bailey (1995). The form of the models can easily be adapted for application in broader geographic regions and species groups than those found in Minnesota (Lessard et al. 2000, Lessard *in press*).

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