Partitioning the Uncertainty in Estimates of Mean Basal Area Obtained from 10-year Diameter Growth Model Predictions

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Abstract.—Uncertainty in model-based predictions of individual tree diameter growth is attributed to three sources: measurement error for predictor variables, residual variability around model predictions, and uncertainty in model parameter estimates. Monte Carlo simulations are used to propagate the uncertainty from the three sources through a set of diameter growth models to estimate the total uncertainty in 10-year predictions of mean basal area per unit area for a sample of Forest Inventory and Analysis plots. Response surface methodology is used to partition the total uncertainty by source. Of the three sources, the uncertainty in parameter estimates contributes most to the variance of the estimate of mean basal area per unit area.

The objectives of this study were threefold: (1) to obtain 10-year predictions of mean plot basal area per unit area for a sample of Forest Inventory and Analysis (FIA) plots using diameter growth models; (2) to propagate uncertainty from three sources (measurement error, residual variability around model predictions, and uncertainty in parameter estimates) through the models to estimate the total variance of mean plot basal area per unit area, and (3) to partition the total uncertainty in the mean plot basal area estimates by underlying source.

Methods

The FIA program of the USDA Forest Service has initiated an annual forest inventory system featuring measurement of a proportion of plots each year, 20 percent annually in much of the eastern United States. One approach to calculating annual inventory estimates using data obtained with the new system is to update to the current year data for plots measured in previous years and then base estimates on the updated information for all plots. If the updating procedure is sufficiently unbiased and precise, this approach is nearly as precise as using all plots but without the adverse effects of using out-of-date information. With the latter estimation approach in mind, a set of individual tree, diameter at breast height (d.b.h.) (1.37 m above ground) growth models was constructed and calibrated for use in updating FIA plot information.

The mathematical form of the d.b.h. growth models is:

$$E(\Delta DBH) = \beta_1 \exp(-\beta_2 DBH) DBH^{\beta_3} \exp\left(\sum_{j=4}^{11} \beta_j X_{j-2}\right) \qquad (1)$$

where E(.) is statistical expectation, d.b.h. is annual d.b.h. growth, the ∃s are parameter to be estimated, and the Xs are predictor variables in addition to d.b.h. The additional predictor variables, X_2-X_9, include a suite of tree and plot variables either measured by FIA field crews or calculated from their measurements. Tree variables include d.b.h., crown ratio (CR), and crown class (CC) at the time of the initial inventory. CR is the proportion of tree height that is in the crown, and CC is a measure of a tree's dominance in relation to adjacent trees in the same stand and is coded as follows: 1-open grown; 2-dominant; 3-codominant; 4-intermediate; and 5-overtopped (USDA FS 2001). Plot variables include latitude (LAT) and longitude (LON) of the plot center, plot basal area (BA), and physiographic class (PC). PC is a measure of site soil and water conditions that affect tree growth coded as follows: 3-xeric; 4-xeromesic; 5-mesic; 6-hydromesic; 7-hydric; and 8-bottomland (USDA FS 2001). Plot basal area in trees larger than the subject tree (BAL) is a plot variable but is calculated for each tree. BA and BAL are the sum of cross-sectional areas of live tree boles at breast height and are scaled to a per unit area basis. Details regarding calibration of model (1) are discussed by Lessard (2001).

The Annualized Inventory Database

An annualized 11-year database of plot and tree variables was constructed using the methodology described by McRoberts (2001) to provide a basis for estimating model prediction uncertainty and the total uncertainty of mean plot BA esti-

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mates. The database was constructed using measurements of forested FIA plots in Michigan, Minnesota, and Wisconsin in Bailey's eco-province 212 (Bailey *et al.* 1994) for the two most recent USDA periodic inventories in each state (Michigan 1979: Spencer and Hahn 1984; Michigan 1993: Leatherberry and Spencer 1996; Minnesota 1977: Spencer 1982; Minnesota 1990: Miles *et. al.* 1995; Wisconsin 1983: Raile 1983: Wisconsin 1996; Schmidt 1998). Because special analyses were necessary to estimate the uncertainty in the d.b.h. growth model parameters, the data were restricted to plots that included only the four most commonly occurring tree species on FIA plots in eco-province 212: red pine, jack pine, balsam fir, and quaking aspen. Thus, if any tree on a plot was any other species, the data for that plot were excluded from the database. The resulting database included information for 2,900 trees on 185 plots.

Beginning with the year 0 annual database values, the models were used to predict d.b.h. growth to obtain d.b.h. estimates each tree for years 1-10. Values of all predictor variables dependent on d.b.h.s were recalculated each year based on the d.b.h. predictions for that year. Estimates of mean plot BA and the standard error of the mean were calculated using stratified estimation (Cochran 1977) where the strata are defined by quartile categories of plot BA, and plots are assigned to strata on the basis of the year 0 plot BA. Estimates of mean plot basal area and the standard error of the mean obtained using this procedure were designated the MODEL estimates. As a standard for comparing the MODEL estimates, estimates of mean plot basal area and the standard errors of the means were calculated each year using the data in the annualized database with the same stratified estimation techniques and were designated the ANNUAL estimates.

Uncertainty in Model Predictions

Uncertainty in d.b.h. growth model predictions was attributed to three sources: uncertainty in values of predictor variables due to measurement errors, residual variability around model predictions, and uncertainty in model parameter estimates. Because of their minimal distributional and linearity requirements and because they produce reliable estimates of model prediction distributions, Monte Carlo methods were used to estimate the total uncertainty in predictions from the growth models and to propagate the uncertainties to the mean plot BA estimates. Before the simulations could be implemented, uncertainty had to be quantified for the underlying sources: measurement error for tree- and plot predictor variables, residual variability, and uncertainty in parameter estimates.

Uncertainty in Predictor Variables.—Distributions for measurement errors for the tree predictor variables were obtained from the literature. McRoberts *et al.* (1994) reported the results of a study in which 9-10 FIA field crews independently measured the same plots. They estimated a curve for describing the standard deviation of d.b.h. measurements as a function of mean d.b.h. They also reported that the distribution of ocular estimates of CR as a percentage in the 0-1 range often deviated ±0.3 around the median crew estimate. Nichols *et al.* (1991) reported that when crews returned to plots later in the same growing season to obtain second ocular estimates of CC, 80 percent of estimates were unchanged while the remaining 20 percent were evenly distributed in the two adjacent classes. Although BA and BAL are plot variables, their estimates are based on individual tree d.b.h. measurements and are also subject to d.b.h. measurement error. Uncertainty in BA and BAL was simulated by using d.b.h. measurements that incorporated simulated measurement error. Finally, because of the nonuniformity of plot soil, topographic, and vegetation conditions, PC is also subject to uncertainty due to sampling variability. However, because no empirical estimates of the sampling variability for PC were available, no uncertainty in the measurement of this variable was considered. In addition, no uncertainty was considered for the LON and LAT predictor variables.

Residual Variability.—Estimates of residual variability were obtained as by-products of calibrating the models. Residuals were assumed to follow a Gaussian distribution but with heterogeneous variances. The standard deviations of the distributions of residuals were found to be adequately described as:

$$E\left[\ln\left(\hat{\sigma}_{res}\right)\right] = \alpha_1 + \alpha_2 \ln\left(\Delta D\hat{B}H\right) \qquad (2)$$

where E(.) denotes statistical expectation, σ_{res} is the sample estimate of Φ_{res}, and $\Delta D\hat{B}H$ is predicted diameter growth from the models.

Uncertainty in Model Parameter Estimates.—Using the distributions of residual variability as previously described, distributions of model parameter estimates were obtained using a four-step Monte Carlo procedure:

1. The parameter estimates obtained from calibrating the models were used with the growth models (1) to predict d.b.h.; using predicted d.b.h., and (2), a residual was randomly selected and added to each prediction to simulate an observation of d.b.h.
2. Simulated measurement errors for d.b.h., CR, CC, and PC were obtained by randomly selecting from the appropriate distributions and adding them to the observed values to obtain simulated observations of these predictor variables; BA using the simulated d.b.h. observations was calculated for each plot, and BAL was calculated for each tree on each plot.
3. Using the simulated observations of d.b.h. from Step 1 and the simulated observations of the predictor variables from Step 2, the models were recalibrated, and the resulting parameter estimates recorded.
4. Steps 1-3 were repeated 250 times to construct a distribution of simulated parameter estimates.

Uncertainty in Model Predictions.—Estimates of mean plot BA and the standard error of the mean were obtained using a four-step Monte Carlo procedure:

1. Year 0:
   a. Each simulation was initiated by simulating measurement of all plots by adding the year 0 observed values of d.b.h., CR, and CC in the annualized database and simulated measurement errors obtained by randomly selecting values from the appropriate distributions;
   b. BA for each plot and BAL for each tree on each plot were calculated using the simulated d.b.h. observations;
   c. Mean plot BA and the standard error of the mean were calculated;
   d. A set of model parameter estimates was randomly selected from the distribution for each species.
2. Subsequent years:
   a. Current year d.b.h. for each tree was calculated as the sum of previous year's d.b.h., the model prediction of d.b.h., and a residual randomly selected from a Gaussian distribution using predicted d.b.h. and [2];
   b. BA for each plot and BAL for each tree on each plot were calculated using the simulated d.b.h. observations;
   c. Mean plot BA and the standard error of the mean were calculated and recorded;
3. Step 2 was repeated 10 times to obtain estimates of mean plot BA and the standard error of mean for years 1-10.
4. Steps 1-3 were repeated 250 times to obtain distributions of estimates of mean plot BA and the standard error of the mean for each year.

For this study, each simulation was considered a separate, independent imputation. Rubin (1987) advocates multiple completions of data sets via imputation to allow assessing the uncertainty in imputed variables and to protect against extreme results and further recommends the separate estimates be combined as follows:

$$\overline{V} = \frac{1}{m}\sum_{k=1}^{m}\overline{V}_k \tag{3}$$

and

$$\mathrm{Var}(\overline{V}) = \frac{1}{m}\sum_{k=1}^{m}\mathrm{Var}(\overline{V}^k) + \frac{m+1}{m}\sigma^2_{\overline{V}} \tag{4}$$

where & $\overline{V}^k$ and $\mathrm{Var}(\overline{V}^k)$ are the stratified estimates of the mean plot BA and the variance of the mean, respectively, for the kth simulation, and $\sigma^2_{\overline{V}}$ is the variance among the separate estimates of mean plot BA. For this study, m=250, far greater than the m=2 or m=3 found to be adequate in unrelated studies by Rubin and Schenker (1986).

Partitioning Uncertainty

The goal in partitioning uncertainty is to quantify the contributions of uncertainties from individual sources to the uncertainty of the estimate of interest. For this study, the total variance of the model-based estimates of mean plot BA for year 10 was partitioned with respect to uncertainty from three aggregated sources: (1) measurement error, (2) residual variability around d.b.h. growth model predictions, and (3) uncertainty in parameter estimates. The uncertainties from all sources were aggregated into these three sources; i.e., measurement errors for all variables were aggregated into the single source, measurement error; residual variabilities for all species were aggregated into

the single source, residual variability; and uncertainties in all parameter estimates were aggregated into the single source, uncertainty in parameter estimates. The uncertainties for individual sources are incorporated into the simulations separately, but their contributions to the total uncertainty of the BA estimates are combined within their respective aggregated sources.

Two approaches to partitioning uncertainty are intuitive. First, the contribution to uncertainty of a single aggregated source may be estimated as the difference between the total uncertainty obtained when the uncertainties for that aggregated source are incorporated and the total uncertainty obtained when no uncertainty from any source is incorporated. This approach is denoted NONE+1. Second, the contribution of a single aggregated source may be estimated as the difference between the total uncertainty when the uncertainties for all sources are incorporated and the total uncertainty when uncertainties for all sources except the aggregated source of interest are incorporated. This approach is denoted TOTAL-1. Estimates of the contributions of individual sources obtained using the NONE+1 and the TOTAL-1 approaches are frequently biased. The bias may be seen by comparing the sums of the estimates of the contributions of all aggregated sources obtained using the NONE+1 and the TOTAL-1 approaches to the difference between the total uncertainty when uncertainties for all aggregated sources are incorporated and the total uncertainty when no uncertainty for any source is incorporated. If the estimates of the contributions from the individual sources are unbiased, the former sums should equal the latter difference. Typically they are not equal when using the NONE+1 and TOTAL-1 approaches. The bias is attributed to lack of independence among the effects of individual sources of uncertainty inherent in the simulation process.

An approach that produces independent estimates of the contributions to total uncertainty by aggregated source is based on response surface methodology (Myers 1971, Khuri and Cornell 1996). With this approach, small-order polynomials are used to describe the relationship between levels of uncertainty for underlying sources and the uncertainty of the estimate of interest. If estimates of total uncertainty are obtained for a factorial arrangement of the levels of uncertainties for the underlying sources and coded through orthogonal transformations, then a response surface may be constructed using orthogonal polynomials that produces uncorrelated coefficient estimates for first-order variables.

For each of the three sources of uncertainty, three levels of uncertainty were considered: the first level incorporated uncertainties for all individual sources corresponding to the standard deviations of the distributions of uncertainty for those sources; the second level simultaneously incorporated uncertainties for all individual sources corresponding to half the standard deviations; and the third level corresponded to no uncertainty from any component source. For the measurement error of predictor variables, the standard deviations were those obtained from the literature, and for residual variability, the standard deviations were calculated from (2). For model parameter estimates, uncertainties for the first level were incorporated in the simulations by randomly selecting from the simulated distributions of parameter estimates. For the second level, random selections were made from the simulated distributions, the deviations of these selections from the means of the distributions were calculated, and then half this deviation was added to the mean. For the third level, the means of the simulated distributions were used. Within each source, the combinations of levels of uncertainty for the individual sources are limited to three: simultaneous use of the full standard deviations for all component sources, simultaneous use of half the standard deviations for all component sources, and no uncertainty for any component source. Thus, 27 sets of simulations were conducted, one for each of the 27 combinations resulting from the three levels of uncertainty for each of the three sources.

The levels of uncertainty for each aggregated source were transformed to facilitate describing the total uncertainty of the mean plot BA estimates using orthogonal polynomials. For each aggregated source, Φ_{max} represented the first level corresponding to the full standard deviation, Φ_{min} represented the third level corresponding to no uncertainty, and Φ represented an arbitrary level. Orthogonal transformations were then applied using the coding formula of Khuri and Cornell (1996):

$$\sigma' = \frac{2\sigma - (\sigma_{min} + \sigma_{max})}{\sigma_{min} - \sigma_{max}} \tag{5}$$

where Φ and Φ' were the untransformed and transformed codings, respectively. Although the standard deviations of the distributions of uncertainties for the individual sources differed, the transformed codings of the three levels of the uncertainties were the same for all individual sources: $\Phi'=1$ for the first

level, $\Phi'=0$ for the second level, and $\Phi'=-1$ for the third level. Thus, the three common values, ($\Phi'=1$, $\Phi'=0$, $\Phi'=-1$) were used to describe the levels of uncertainty for an entire aggregated source. Orthogonal polynomials were based on the three values for each of three predictor variables, Φ'_1, Φ'_2, and Φ'_3, one for each source. These 27 combinations of values of 1, 0, and -1 for the three sources constituted an orthogonal design. Thus, $\mathrm{Var}(\bar{Y})$ was described using orthogonal polynomials ex-pressed using linear, quadratic, and two-way interaction terms as:

$$\mathrm{Var}(\bar{Y}) = \beta_0 + \sum_{i=1}^{3}\beta_i\sigma'_i + \sum_{i=1}^{3}\beta_{ii}\sigma'_i + \sum_{i=1}^{3}\beta_{ij}\sigma'_i\sigma'_j + \varepsilon \qquad (6)$$

where $\mathrm{Var}(\bar{Y})$ is the estimated variance of mean plot BA obtained from (4), Φ'_i is the predictor variable associated with the ith source of uncertainty, $\exists_0$ is the intercept coefficient, the $\exists_i$s are linear coefficients, the $\exists_{ii}$s are quadratic term coefficients, and the $\exists_{ij}$s and interaction term coefficients.

Although the estimates of the $\exists_i$s are uncorrelated with each other because of the orthogonal design, the estimate of any $\exists_i$ is not uncorrelated with the estimate of $\exists_0$, the estimates of the $\exists_{ii}$s, or the estimates of the $\exists_{ij}$s. Nevertheless, the coefficient estimates may be used to estimate the contribution to the total variance of the estimate of mean plot BA from the three aggregated sources and to partition the variance with respect to the contributions from those sources. The total uncertainty in the mean plot BA estimates was calculated using (6) with $\Phi'_1=\Phi'_2=\Phi'_3=1$, which corresponds to the maximum or first level of the uncertainty for all component sources. The portion of the total uncertainty attributed to the ith aggregated source was estimated by setting $\Phi'_i=-1$, the minimum or third level of uncertainty for that source, and setting $\Phi'=1$, the maximum level, for the other aggregated sources, calculating the uncertainty of the mean plot BA estimate using (6), and subtracting the result from the total uncertainty estimate. This approach is analogous to the NONE+1 approach, except that it is based on predictions from (6) rather than simulated estimates. An approach analogous to the TOTAL-1 approach was also used. The estimate of uncertainty remaining after the contributions from each of the three sources have been estimated was attributed to natural variability among plots, can only be reduced by using techniques such as stratified estimation, and was designated sampling variability. Because the estimates of the contributions of aggregated sources are independent, the NONE+1 and the TOTAL-1 approaches produce identical results when used with a linear model, but do not necessarily produce identical results when the model includes quadratic and/or interaction terms.

Results

The adequacy of the 250 simulations was checked by evaluating the stability of estimates of means and standard errors of means. Plots were ordered by their variability over simulations in these coefficients of variation, and a graph of coefficients of variation versus simulation for the four plots with the greatest variability revealed that stability was achieved by approximately 100-150 simulations. Therefore, 250 simulations were deemed adequate to evaluate uncertainty.

The MODEL mean plot BA estimates tracked the ANNUAL means closely, while the MODEL standard errors were only slightly greater than the ANNUAL standard errors (table 1). The Wilcoxon Signed Ranks test (Conover 1980) detected no statistically significant differences ($\alpha = 0.05$) between the ANNUAL and the MODEL estimates of mean plot BA. The slight differences in the standard error estimates indicate that the additional uncertainty due to using the growth model predictions to predict d.b.h. introduced little additional uncertainty into the standard errors of the 10-year mean plot BA estimates.

Table 1.—*Comparisons of ANNUAL and MODEL estimates of mean plot BA*

Year	ANNUAL		MODEL	
	Mean	SE	Mean	SE
0	6.6413	0.2235	6.6413	0.2235
1	7.4136	0.2418	7.4574	0.2386
2	8.2129	0.2765	8.4553	0.2869
3	9.1482	0.3129	9.9568	0.3445
4	10.0728	0.3607	10.7690	0.4253
5	11.1123	0.4261	12.0680	0.5256
6	12.2704	0.5028	13.5109	0.6404
7	13.6350	0.5970	15.1985	0.7797
8	15.0879	0.6925	16.9822	0.9266
9	16.6195	0.7965	18.8644	1.0884
10	18.3350	0.9086	20.9654	1.2566

Table 2.—*Partitions of the variance of mean plot BA*

Aggregated source	Method			
	Observed		Response surface	
	NONE+1	TOTAL-1	LINEAR R^2=0.8902	QUADRATIC R^2=0.9999
Sampling variability (SV)	0.8902	0.8902	0.7309	0.8869
Measurement error (ME)	0.0146	0.0409	0.0227	0.0227
Residual variability (RV)	0.0043	0.0032	0.0008	0.0008
Parameter uncertainty (PU)	1.5462	1.5906	1.5682	1.5682
Subtotal 1 (ME+RV+PU)	1,5651	1.6347	1.5918	1.5917
Subtotal 2 (TOT-SV)	1.6030	1.6030	1.5918	1.5917
Total (TOT)	2.4932	2.4932	2.3227	2.4786

Of the three sources of uncertainty considered, parameter uncertainty made the greatest contribution to total uncertainty, while the contributions of measurement error and residual variability were negligible (table 2). A comparison of the Subtotal 1 and Subtotal 2 values for the NONE+1 and TOTAL-1 approaches revealed the bias inherent in the estimates of the contributions of the aggregated sources. Although the differences were not great, the LINEAR and QUADRATIC response surface models produced values that were nearly identical. Due to orthogonality, this result was expected and necessary for the LINEAR model but was an unexpected positive result for the QUADRATIC model. Based on the large R^2 =0.9999 for the QUADRATIC model, the estimates of the contributions of the aggregated sources were considered reliable.

Conclusions

Two conclusions may be drawn from this study. First, the model-based d.b.h. prediction technique had only a slight negative impact on the total uncertainty of 10-year predictions of mean plot BA. Second, among the uncertainties propagated through the model, uncertainty in the model parameter estimates made the greatest contribution to the total uncertainty in the mean plot BA estimates. Admittedly, a complete prediction system also requires techniques for predicting the survival, regeneration, and removal of trees, components that were not considered in this study.

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