A STAND TABLE PROJECTION MODEL FOR NATURAL HARDWOOD STANDS IN RED RIVER BOTTOMS.[1/]

DENNIS L. MENGEL[2/]

Keywords: Leslie matrix, Markov chain, even-aged, mixed species

INTRODUCTION

This research investigates the application of a Leslie matrix growth model to even-aged, mixed species hardwood stands found in Red River Bottoms of the Southeastern U. S. Coastal Plain.

Models, which project stand diameter distribution using a matrix of probabilities and current diameter distribution, are a form of whole stand/distance independent models. Matrix models have certain advantages in that they are relatively simple to develop and use. Matrix models are also easy to understand by users without a mathematical or statistical background. The major limitation is the matrix model will not project a stand for a time period that is not an integer multiple of the remeasurement period used to develop the matrix. However, Harrison and Michie (1985) have created a 1-year transition matrix that overcomes this disadvantage.

Leslie (1945) developed the first matrix model for population dynamics, which has been applied to selection forests by Usher (1966) and to uneven-aged mixed hardwood stands by Bruner and Moser (1973) and Buongiorno and Michie (1980).

METHODS AND MATERIALS

The Leslie Matrix is used in a stochastic process known as a Markov chain. To model a Markovian system it is assumed that 1) given the current state, the past states have no influence on the future states (Markov property) and 2) the transitional probability of moving from one state to another remains constant with respect to time (stationary property). States are here defined as 1-inch diameter classes. Transition probabilities are the probabilities of a tree remaining in one diameter class, moving to another or dying. This suggests that the future diameter distribution only depends on the current diameter distribution and the probability of a tree moving from one diameter class to another.

The Markov process can be described by the following matrix formula:

$$XP = Y$$

where:

X = a (1xn) vector describing the current diameter distribution with n being the number of 1-inch diameter classes.

P = a (nxn) upper triangular matrix containing the transition probabilities

Y = a (1xn) vector describing the new diameter distribution.

Transition probabilities are empirically estimated as the frequency of tree movement from one diameter class to another over one growth period. The sum of the probabilities for a tree moving to another state, including mortality or being in the same state, must equal 1. For example, if there were 100 2-inch trees at the beginning of the measurement period and 5 years later 50 of those remained in the 2-inch class, the probability of a 2-inch tree staying in the 2-inch diameter class would be 50/100 or 0.5. All the probabilities, including mortality are calculated in this manner and combined to form the transition matrix. Ingrowth is a fixed value per diameter class based on average ingrowth values determined from the remeasurement data.

The data used to fit this model were collected on 14 permanent growth plots maintained by the North Carolina State University Hardwood Research Cooperative. The plots were located throughout the Southeastern Coastal Plain in even-aged, fully stocked stands. Stand age ranged from 10 to 55 years with 175 to 1290 trees/acre. The plots were remeasured at 5-year intervals with 1 to 3 measurements/plot depending on when the plot was established.

Model validation is an important part of the model development process. While validation is not a guarantee that a model will perform adequately under all stand conditions, it can show how a model will perform given certain stand characteristics. Past research on matrix model development has been hampered by a lack of data to test the model with. For model validation purposes in this study, the data set was divided into 2 data sets, with 7 plots each, based on age and geographic location. A matrix was developed with one set and tested against the other set. Each plot in the test data set was projected using the matrix and the observed and predicted distributions were compared using Chi-square. The same analysis was repeated with the data set roles reversed. The data sets were then combined to fit the final matrix.

1/ Poster presented at the Sixth Central Hardwood Forest Conference, Knoxville, TN, February 24-26, 1987.

2/ Forest Biometrician with the North Carolina State University Hardwood Research Cooperative, Box 8002, Raleigh, NC 27695.

Table 1. Matrix of 5-Year Transition Probabilities

Initial DBH (Inches)	DBH 5 Years Later (Inches)																														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	M
1	.51	.22	.03	.02	.01	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.21
2		.56	.13	.04	.01																										.26
3			.51	.14																											.35
4				.56	.11																										.33
5					.54	.23	.02	.01	.01	.01																					.18
6						.53	.29	.05	.01																						.12
7							.51	.35	.04	.02			.01																		.07
8								.50	.36	.05	.01				.01																.07
9									.33	.47	.08	.01																			.11
10										.41	.44	.10																			.04
11											.26	.55	.11																		.08
12												.39	.36	.17	.02																.06
13													.30	.58	.07	.02															.03
14														.25	.59	.13															.03
15															.21	.58	.11			.05											.05
16																.17	.59	.12													.12
17																	0	.89	.11												
18																		.20	.60	.20											
19																			0	.75	.25										
20																				.20	.20	.20	.20	.20							
21																					.33	.67									
22																						.50	.50								
23																							0	1.00							
24																								0	1.00						
25																									1.00						
26																									.00	.00	.00	.00	.00	.00	
27																										.00	.00	.00	.00	.00	
28																												.50	.00	.50	

RESULTS AND DISCUSSION

Table 1 presents the final 5-year transition matrix based on the complete data set. The probabilities indicate that the majority of the trees less than 9 inches dbh remain in the same diameter class or die. This behavior would not be reasonable for young vigorously growing stands. The data used to calculate the probabilities had very few young plots which contributed to the above trend. Therefore, extrapolation of this matrix outside the range of data used to fit it would not be recommended.

Chi-square comparisons between observed and predicted survivor distributions indicate that the distributions were similar ($\alpha = 0.05$). Visual examination of the observed vs. predicted distributions showed good correlation of pulpwood size trees (dbh $\geqslant$ 4 in.). The accuracy of prediction decreases for trees less than 4 inches dbh. This indicates the difficulty in accounting for the wide variation in mortality and ingrowth of small trees by diameter alone. To correct the shortcomings of this current model, future work will concentrate on developing numerical models for describing ingrowth and mortality. Methods to incorporate density into the projection will also be investigated.

LITERATURE CITED

Bruner, H. D. and J. W. Moser. 1973. A markov chain approach to the prediction of diameter distributions in uneven-aged forest stands. Can. J. For. Res. 3:409-417.

Buongiorno, J. and B. R. Michie. 1980. A matrix model of uneven-aged forest management. Forest Sci. 26:609-625.

Harrison, T. P. and B. R. Michie. 1985. A generalized approach to the use of matrix growth models. Forest Sci. 31:850-856.

Leslie, P. H. 1945. On the use of matrices in certain population mathematics. Biometrika 33:183-212.

Usher, M. B. 1966. A matrix approach to the management of renewable resources, with special reference to selection forests. J. Applied Ecol. 3:355-367.