EVALUATION OF A COMMON VOLUME EQUATION FOR SCARLET OAK AND BLACK OAK IN THE MISSOURI OZARKS[1/]

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Abstract.--The accuracy and precision of a volume equation whose coefficients have been estimated for several states was evaluated for the Missouri Ozarks using scarlet and black oak trees. Trees were felled, diameter measured along the length of the stem, gross cubic foot volumes calculated from these measurements, and the sawlogs extracted from these trees sawn into boards which were then tallied by tree. Using the volume equation, cubic foot and board foot volumes were then estimated and compared to the true volumes. The equation tended to underestimate cubic foot volume and overestimate board foot volume with estimates of either being within approximately 40% of the true values. Underestimation of cubic foot volume increased with tree dbh as did misestimation of board foot volume. Much of the imprecision and inaccuracy of the equations was apparently due to misestimation of cull in standing trees by field crews, and by the limited predictive ability of the original composite volume tables from which were derived tree volumes estimates used to fit the equation(s).

Keywords: volume equation, Missouri, Ozarks.

INTRODUCTION

The estimation of volume in standing trees is one of the most basic problems in forest measurements. Some geographic regions of the United States have well-developed volume equations and/or tables which are relatively uniform and consistent throughout a region. This is the case, for example, in the southeastern United States where large-scale plantation forestry is commonly practiced -- the uniformity of plantations and large contiguous forest blocks facilitate the development of standardized volume equations (Bryan and McClure 1962, Vimmerstedt 1962, Bennett et al. 1959). In other regions such as the northeastern United States, there is little consistency in volume equations and tables throughout the region and existing tables and equations vary widely in accuracy and precision (Young and Hodsdon 1966, Honer 1965, Bartoo and Hutnik 1962). This is due partly to a lack of uniform forestry practices within the region and also to a highly fragmented ownership pattern for forest land (Kingsley and Birch 1977).

Another geographic area that suffers somewhat from a similar problem is the central hardwood region of the midwest United States. One state in this region that epitomizes the problem is Missouri. Missouri is a predominantly agricultural state whose forest land tends to be owned by non-industrial private individuals who own relatively small noncontiguous blocks (Lewis 1979). Landowners often do not conduct forest management on their timberland either because they view themselves as agriculturalists only rather than forestland managers also, and/or because they are unaware of the value of their timber resource. This situation contributes to the lack of consistency among volume tables throughout Missouri and also geographically identifiable areas within Missouri -- agriculturalists do not have access to reliable volume tables to

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determine the value of their resource and therefore do not conduct forest management, and because they do not manage their forests, they are not concerned with the reliability of existing tables. Efforts by the authors of this paper to obtain generally accepted volume tables/equations for Missouri and/or regions within Missouri produced either a complete lack of volume tables/equations for a region, or a remarkable diversity in existing tables/equations whose origin was unknown.

The U.S.D.A. North Central Forest Experiment Station has attempted to overcome this problem for a number of states within the midwest. A single volume equation form has been fitted for Wisconsin (Hahn 1973), Iowa (Hahn 1976), and Missouri (Hahn 1975), among others. The form of the equation is:

$$\text{Volume} = (A * SI) * B^{C^{(D * dbh)}} \qquad (1)$$

where: Volume is tree volume,
A, B, C, and D are regression coefficients,
SI is site index (height (ft) at base age 50 years, and
dbh is tree diameter breast height (inches).

Equation fitting was done similarly for all states. Diameter and total height data were recorded on numerous trees (10 000+) as part of the Forest Survey of each state. Gross cubic foot underbark volumes and net board foot volumes (International 1/4") of these trees were estimated from composite volume tables for the Lake States (Gevorkiantz and Olsen 1955). These estimates were further modified by correcting for differences in bark thickness and for estimates of cull volume made by field personnel. These data were then stratified by species group, geographic regions within a state, and as "desirable" trees (live trees with no serious defects which would be favored in silvicultural operations) or "acceptable" trees (all others). Using site index data also collected during the Forest Survey, coefficients of the volume equation were fitted for each stratum. To facilitate the use of these equations, average site index was also estimated for each stratum.

This method has potential benefits because it utilizes a single volume equation form for an entire state, and is also somewhat localized by having coefficients estimated for individual regions within a state. However, there are a number of potential sources of imprecision and inaccuracy (bias) in the resulting equations for Missouri and other midwestern states. First, the tree volumes used to fit the equations were not actually measured but were estimated from tables based on limited data and/or data summaries. Moreover, these original tables were derived for the Lake States rather than Missouri. Second, the utility of average site index rather than an actual measured value is unknown. And third, even if neither of the preceding factors cause inaccuracy and/or imprecision, it is unknown whether or not the equation form chosen is suitable for estimating tree volume.

The purpose of this paper is to evaluate the suggested volume equation to determine its accuracy, precision, and limitations for a single area in Missouri. Though the present study was conducted in a highly specific situation, the results should be of interest to other situations for which a similar equation has been fitted.

DATA

Data were collected on University Forest -- a 7 000 acre forest owned by the University of Missouri and administered by the School of Forestry, Fisheries, and Wildlife. In 1953, a replicated block design thinning experiment was established which has been monitored approximately every 5 years since that time. In 1985, it was decided to harvest that study as part of a forest economics Ph.D. dissertation. For that study, it was decided that, if possible, 12 scarlet oak (*Quercus coccinea* Muenchh.) trees and 12 black oak (*Quercus velutina* Lam.) trees -- the predominant species in the experiment -- would be randomly selected for harvest on each plot. Though 12 scarlet oak trees were present on each plot, it was not usually possible to sample 12 black oak trees per plot. In the latter case, all black oaks on a plot were selected for harvest. This resulted in a total of 162 trees being sampled -- 90 scarlet oak and 72 black oak trees.

Trees were felled and sectionally measured: total height, crown height, and merchantable height were recorded, and diameter overbark was recorded at stump height, breast height, 2.5, 10.0, 17.3 feet above ground, and height steps of approximately 7 feet above 17.3 feet. Four bark thickness measurements were also recorded at each point where diameter was measured. In the case of trees forking into 2 or more stems, a single dominant branch was identified and measured from the fork upwards. Trees were then bucked into sawlogs which were scaled (International 1/4") and skidded to the sawmill at University Forest where they were sawn into boards. The origin of each board was noted and the total lumber outturn from each tree tabulated. Using the conic integral, the sectional measurements were expanded to provide a "best estimate" of net cubic volume

underbark. These data were considered the true cubic foot volumes of the trees just as the sawn lumber was considered the true board foot content of each tree. Note that because cubic foot volume was not determined through water displacement, the cubic foot volumes are best estimates rather than actual true volumes. Note also that slightly different board foot volumes would have been obtained under different sawing and harvest schemes. However, these data were as good or better than the data used to produce the original composite volume tables from which the equation under study was fitted. Hence, it was felt that these data provided an extremely useful means to evaluate both the cubic foot and board foot volume equation and the applicability of the original tables to Missouri.

PROCEDURE

Because the trees harvested were originally given preferential silvicultural treatment as part of the original thinning experiment, all were considered "desirable" rather that "acceptable" trees. Using coefficients for the Eastern Ozarks -- the geographic unit where University Forest lies -- both cubic foot and board foot volumes were generated for all trees. Because both scarlet oak and black oak are in the same species group identified by the author of the volume equation, the same coefficients were used for both species. Results are tabulated and presented for each species individually, however. Volumes were generated using both the average site index suggested by the author of the volume equation, and also using site index estimated from measurements taken on the study area.

True values were compared to respective estimated values and residuals calculated as "estimated minus actual" so that a negative residual indicates underestimation. Each set of residuals was analyzed three ways. First, the mean residual and $\underline{t}$ statistic were calculated to test if the mean residual was significantly different from zero -- i.e. if significant bias was present. The mean residual also indicates whether the tendency of the equation is to underestimate or overestimate volume. However, because $\underline{t}$ may produce misleading results if precision is poor, a measure of precision suggested by Freese (1960) was also calculated at the 95% confidence level. This statistic is here termed the "maximum expected residual" (MER). The MER for each species and predictive methodology was also divided by the mean measured volume for all trees of a particular species and multiplied by 100 to indicate the "percent precision" -- i.e. within what percent of true the volume equation predicted. This percent precision may be viewed as the "average" percent deviation of predicted from actual values. Third, residuals were graphed against dbh and correlated with dbh to determine if the degree of underestimation or overestimation was related to tree size.

RESULTS AND DISCUSSION

Table 1 presents the results of the residual analyses for gross cubic foot volume. As mentioned, residuals were calculated using volumes derived from equation 1 using both the average site index supplied by Hahn (1972) and the actual site index measured on the site, and also for volumes from composite tables in the original publication (Gevorkiantz and Olsen 1955).

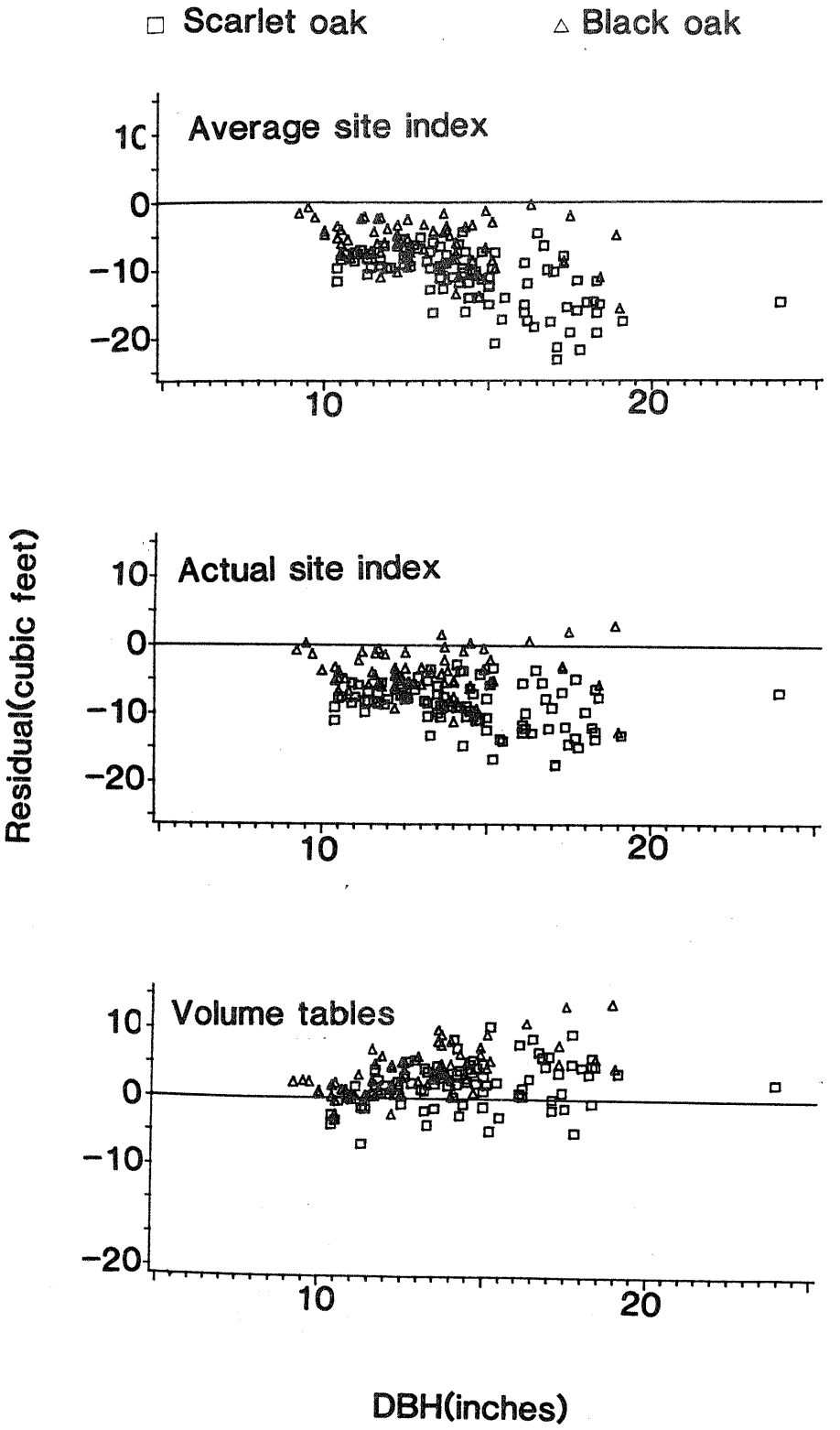
It is apparent that for both species, whether using average or actual site index, volumes were significantly underestimated by the volume equation and the precision of estimates was somewhat poor. Cubic foot volumes were most poorly estimated using average site index -- these estimates had the greatest absolute mean bias, $\underline{t}$ values, maximum expected residual, and percent precision. Estimates were improved slightly by using the actual site index values (which were generally higher than the suggested average site index values) -- absolute mean bias decreased as did the $\underline{t}$ value and the maximum expected residual. However, even when using the actual site index values, significant bias resulted and the maximum expected residual was still approximately 40% of actual. A volume equation that is only within 40% is unacceptable for many applications.

Table 1.--Results of residual analysis for cubic foot volumes.

Residual Derivation	Mean Resid	t value	MER	% Precision
Scarlet Oak (n=90)	(ft^3)		(ft^3)	
Average site index	-11.2	-25.0**	21.1	59
Actual site index	-8.9	-25.4**	16.7	47
Gevorkiantz and Olsen	2.1	5.8**	6.9	19
Black Oak (n=72)				
Average site index	-6.2	-16.0**	12.3	49
Actual site index	-4.5	-12.0**	9.5	38
Gevorkiantz and Olsen	3.8	9.3**	8.9	35

** - significantly different from zero at $p \leq 0.01$.

Figure 1. Residuals for cubic foot volume.



Though the accuracy and precision of volume estimates for each species initially appears quite different with the scarlet oak volume estimates appearing worse -- mean residuals and maximum expected residuals are nearly double for scarlet oak compared to black oak -- this is not actually the case. In the stands studied, black oak had consistently been relegated to the understory resulting in trees considerably smaller than the scarlet oak trees as measured by both dbh and total height. Because the variance of estimates from any volume equation tends to increase with tree size, it is to be expected that the larger (scarlet oak) trees would be more poorly estimated than the smaller (black oak) trees. Note that the percent precision -- maximum expected residual relative to average tree volume for a species -- is approximately the same leading to the conclusion that volume estimates for the two species relative to size are approximately the same.

Just as the fitted volume equation performs somewhat poorly, the analysis of residuals for volumes from the original composite volume tables show that these tables do not perform extremely well for cubic foot volume either. The volume tables significantly overestimated (positive mean residual) tree volumes and the maximum expected residual was approximately 27% of the actual mean volume for both species. However, the volume table estimates did show a marked improvement over estimates from the volume equation though it was surprising that the direction of bias differed between the equation and the original tables. This is most likely due to the volumes used to fit the equation being a function of not only the original composite volume tables, but also a semi-subjective assessment of cull volume by the field personnel who participated in the Forest Survey. It appears that these visual cull estimates were excessive.

One additional result to note from Table 1 is that the original composite volume tables for the Lake States provide more accurate and precise volume estimates of scarlet oak trees than black oak trees. Both the mean bias and percent precision for scarlet oak trees were considerably less than for black oak trees. Because the original composite volume tables were based on specific assumptions of taper and bark thickness, the greater accuracy and precision of scarlet oak estimates implies that scarlet oak trees better conform to the assumptions of the table than do black oak trees.

Figure 1 shows the residuals graphed against dbh for each set of estimates. For both species, but especially scarlet oak, the degree of underestimation appears to increase somewhat with dbh. This was confirmed by correlation analysis which showed that residuals for cubic foot volume estimates from the equation were significantly (0.003) and negatively correlated with dbh. Conversely, for the volumes derived from the original composite volume tables, the misestimation is significantly (0.001) and positively correlated with dbh. This again suggests that the estimates of cull by field personnel were too high, and that this overestimation of cull increased considerably with tree size.

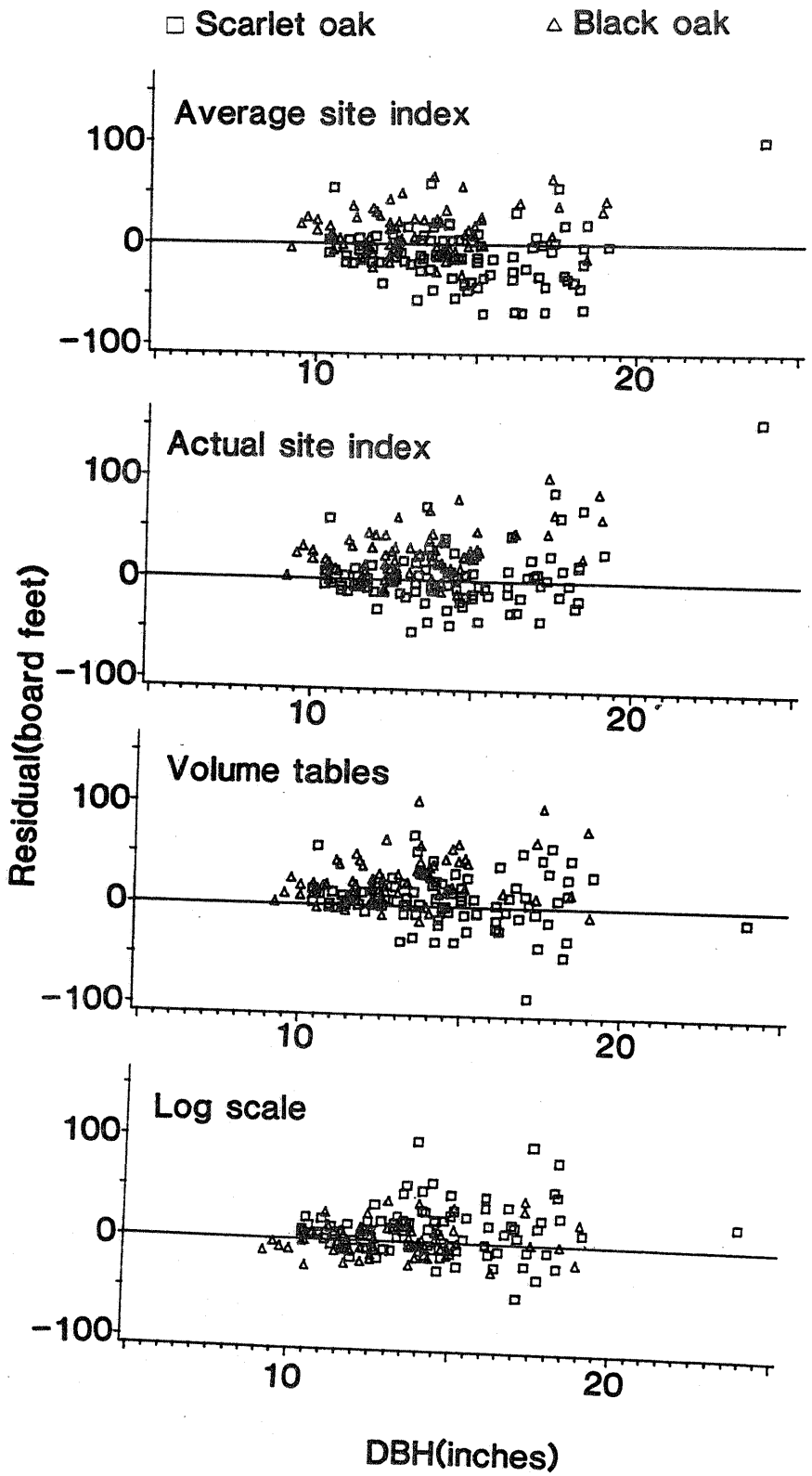
Table 2 shows the results of residual analysis for board foot volume. Results were tabulated in the same manner as cubic foot volume results except that in addition to the estimates used in Table 1, the log scale was also compared to the mill tally. Table 2 suggests that the board foot volume coefficients perform somewhat better than do the cubic foot volume coefficients. Absolute $\underline{t}$ values for board foot volumes are less than for corresponding cubic foot volume estimates suggesting improved accuracy for board foot volume estimates. Further, the percent precision values are lower for board foot volume estimates suggesting improved precision also. Interestingly, using the actual site index to estimate board foot volumes resulted in volume estimates of approximately equal accuracy and precision as estimates from the original composite volume tables. This suggests that, whereas the estimates of cull were too high for gross cubic foot volumes, they were more accurate or slightly underestimated for net board foot volumes. Further, using the actual site index values in equation 1 will yield estimates comparable to those from the original composite volume tables.

Table 2.--Results of residual analysis for board foot volumes.

Residual Derivation	Mean Resid	t value	MER	% Precision
Scarlet Oak (n=90)	(bd ft)		(bd ft)	
Average site index	-11	-3.41**	53	31
Actual site index	5	1.45	54	31
Gevorkiantz and Olsen	7	2.44*	46	27
Log scale	13	4.57**	51	29
Black Oak (n=72)				
Average site index	12	4.66**	42	38
Actual site index	23	8.08**	58	53
Gevorkiantz and Olsen	26	9.49**	60	55
Log scale	2	1.21	27	25

* - significantly different from zero at $p < 0.05$.
** - significantly different from zero at $p \leq 0.01$.

Figure 2. Residuals for board foot volume.



Notably, however, using average site index in this case caused an underestimation of scarlet oak volumes, but an overestimate of black oak volumes. Table 2 also shows that board foot volumes are better estimated for scarlet oak than for black oak as was also the case for cubic foot volumes. This again suggests that scarlet oak better conforms to the assumptions of the original composite volume tables than does black oak.

Another result of interest in Table 2 is the residual analysis for log scale compared to mill tally for a tree. The International Log Rule is known to be a relatively accurate predictor of mill tally (Freese 1973). Consequently, a priori it was expected that the mean residual would not be significantly different from zero, and that the precision of estimates would be better than estimates from either the volume equation or original composite volume tables. While these expected results did occur for black oak, the log scale for scarlet oaks significantly overestimated the mill tally. Moreover, the precision of estimates was only comparable to the estimates obtained from both the volume equation and the original volume tables. Though the cause of this result is uncertain, a likely effect is that this finding is partly responsible for the misestimation of tree volumes by both equation 1 and the tables.

Figure 2 shows the residuals for board feet graphed against dbh. Generally, it does not appear that the degree of bias increases with tree size. An exception to this is both scarlet and black oak volumes estimated from the equation using the actual site index indicating a sensitivity of the equation to site-induced shape differences among trees. Moreover, the correlation coefficient was positive and significant (0.002) for both species indicating that the overestimation of tree volume increases with tree size. This suggests that estimates of cull for net board feet by field personnel became excessively low as tree size increased -- a result which contradicts what was suggested for cubic foot volume cull estimates. A possible explanation is that estimates of cull were based on the lowest portion of the stem, and these cull estimates were actually too low for this portion (where most board feet are located), but too high an "average" for the entire tree. It is uncertain why these results from the volume equation using actual site index were markedly different from the results obtained by generating board foot volume estimates using the average site index. In all cases, imprecision appears to increase with tree size as is often the case with biologic phenomena.

CONCLUSIONS

For the trees examined, the volume equation fitted from tree volumes derived from composite volume tables for the Lake States performs better for net board foot volumes than gross cubic foot volumes. Some of the inaccuracy and imprecision inherent in the volume equation appears to have been due to the original volume tables not being readily applicable to the midwest United States, although there is evidence that these tables were more applicable to scarlet oak trees than black oak trees. Estimates of cull by field personnel used to modify volume estimates from the table appeared inaccurate, but it is uncertain if the inaccuracy was random or systematic. It was apparent, however, that both accuracy and precision of estimates from the equation decreased with increasing tree size. There was no evidence to suggest that the form of the volume equation is inappropriate.

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