A PRELIMINARY COMPARISON OF THREE TREE GROWTH MODELS FOR CENTRAL STATES SPECIES

Stephen R. Shifley[1/]

Abstract.--Three distance-independent models for estimating tree diameter and basal area growth were calibrated and tested for shortleaf pine (*Pinus echinata*), sugar maple (*Acer saccharum*), hickory (*Carya* spp.), black walnut (*Juglans nigra*), white oak (*Quercus alba*), northern red oak (*Q. rubra*), and black oak (*Q. velutina*). These models were the STEMS program diameter growth model, a much simplified version of the Prognosis program basal area growth model, and a model that was developed specifically for this study. Data for calibrating and testing these models came from more than 2,700 remeasured inventory plots in Missouri, Indiana, and Ohio. Each model's predictions of annual diameter growth and annual basal area growth per tree were compared against growth observations from calibration and validation data sets. Averaged over all species, the performance of all three models was similar for the range of species and conditions considered. The STEMS model produced slightly better estimates of annual diameter growth as measured by the mean error in growth estimation and by the square root of the mean squared prediction error. The new model produced slightly superior estimates of annual basal area growth per tree. But for short projection periods (approximately 10 years), differences in the performance of the three models were small in magnitude. Differences among the models in their biological assumptions and their ease of calibration may help to determine the suitability of each model for specific applications.

Keywords: Diameter growth, basal area growth, *Pinus echinata*, *Acer saccharum*, *Carya* spp., *Juglans nigra*, *Quercus* spp., STEMS, PROGNOSIS.

Central States forests are well suited to the application of individual-tree, distance-independent modeling methods (terminology follows Munro (1974)). Stands with mixed species, multiple age classes, non-normal stocking, and other sources of diversity are common throughout the Central States. Individual-tree models project the growth of each tree in a forest stand or on an inventory plot using species-specific coefficients. Thus, such models are not restricted by stand species composition, age structure, or size class. In addition to accommodating a broad range of forest conditions, these models offer great detail and flexibility in reporting projection results. Tree growth projections can be reported individually, aggregated by stand components, aggregated into total stand change, or aggregated into total forest change.

1/ Biometrician, North Central Forest Experiment Station, St. Paul, Minnesota.

An individual-tree growth projection system being tested for use in the Central States is the Stand and Tree Evaluation and Modeling System (STEMS) (USDA Forest Service 1979, Belcher et al. in preparation). The STEMS computer program uses individual tree growth and mortality models to simulate tree and stand dynamics and management. Currently calibrated for 26 Lake States species groups, STEMS has the flexibility to be useful in a variety of applications (see for example Jakes and Smith (1980) or Raile and Smith (1982)). The development of similar models for Central States species will allow the direct application in the Central States of the many techniques, programs, and methods developed for use with STEMS in the Lake States.

The STEMS growth model for Lake States trees is only one of many alternative individual-tree growth models that might be applied to Central States species. Models described by Stage (1973) and Wycoff et al. (1982), or by Dale (1975) represent some other possibilities. All these models require similar information for calibration and application, and they produce similar kinds of output.

Rather than calibrate[2/] one tree growth projection model arbitrarily selected for use with Central States species, I intensively studied three alternative growth models. This paper describes and compares the results of calibrating those three models for seven species.

DATA

A major barrier to developing any system of individual-tree growth projection models is finding an adequate data base to calibrate and test the model. I was fortunate to have the cooperation of many organizations and individuals to provide continuous forest inventory records for this effort (Table 1).

Each inventory plot had been measured two times at an interval of 7 to 12 years. Initial stand characteristics for each plot (e.g. basal area per acre, number of trees per acre, etc.) were computed using all trees on the plot, and this information was appended to each tree record for that plot. Annual tree diameter growth and tree basal area growth at breast height outside bark was computed by dividing growth for the remeasurement interval by the number of growing seasons between measurements. Every fourth plot was then removed from the data base and reserved for model validation.[3/] Then the tree records for the following seven species--shortleaf pine (*Pinus echinata*), sugar maple (*Acer saccharum*), hickory (*Carya spp.*), black walnut

---

2/ Throughout this paper the terms "calibrate" and "calibration" refer to fitting species-specific regression coefficients to a generalized model form.

3/ Throughout this paper "validation data" refers to the 1/4 of the data reserved for testing prior to regression coefficient estimation. In accordance with popular usage, the terms "validate" and "validation" refer to comparing a model's growth predictions with actual growth observed and recorded for trees in the validation data set.

Table 1.--Source of data for growth model analysis.[1/]

Source	Location	Sample type	Years between measurements	Plots	Trees[2/]
			----------No.----------		
Wayne-Hoosier National Forest	Indiana Ohio	U.S.F.S 10-point cluster plot 37.5 Basal Area Factor (BAF) Prism	10	160	3,387
Mark Twain National Forest	Missouri	U.S.F.S 10-point cluster plot 37.5 BAF Prism	7	224	4,947
Bottomland Hardwood Plots	Indiana	U.S.F.S 10-point cluster 37.5 BAF Prism	11	35	2,4051
Missouri State Inventory	Missouri	Single Point Sample 5 BAF Prism	12	1,920	617
Pioneer Forest	Missouri	1/5 Acre Fixed Size Plot	10	417	11,617
Total				2,756	44,619

1/ I gratefully acknowledge the assistance and cooperation of the following organizations and individuals who provided these and other data essential to the conduct of this study and other related studies.

U. S. Forest Service, North Central Forest Experiment Station (A. Jerry Ostrom, Robert Rogers, Ivan Sander, Richard Schlesinger, W. Brad Smith).
Pioneer Forest, Salem, Missouri.
Purdue University (John W. Moser, Jr., W. L. Mills, W. Lloyd Fix).
Tennessee Valley Authority (Robert Gregory, Robert Brooks).
University of Illinois (George Gertner, Dieter Pelz).

2/ Includes cut, dead, ingrowth, and other trees unsuitable for growth model analysis

Table 2.--Mean, minimum, and maximum values for selected calibration data characteristics.

Species	DBH		Plot Basal Area		Mean Stand DBH		Site Index		Trees
	Mean	(Min,Max)	Mean	(Min,Max)	Mean	(Min,Max)	Mean	(Min,Max)	
	(cm)		(m^2/ha)		(cm)		(m at age 50)		No.
Shortleaf pine	20	(3, 59)	15	(2, 30)	15	(3, 33)	18	(8, 30)	1,600
Sugar maple	23	(3, 78)	18	(6, 31)	14	(4, 42)	20	(9, 30)	325
Hickory	18	(3, 70)	15	(2, 39)	13	(3, 28)	19	(8, 30)	2,464
Black walnut	25	(4, 60)	16	(2, 39)	15	(3, 55)	19	(9, 39)	264
White oak	25	(3,109)	16	(2, 32)	15	(3, 57)	18	(8, 30)	3,743
Red oak	29	(3, 89)	15	(3, 32)	15	(3, 41)	18	(5, 30)	1,041
Black oak	25	(3, 94)	16	(2, 32)	14	(3, 55)	19	(8, 30)	3,554
All species	23	(3,108)	16	(2, 39)	14	(3, 57)	19	(5, 30)	12,991

(Juglans nigra), white oak (Quercus alba), northern red oak (Q. rubra), and black oak (Q. velutina)--were separated to analyze and calibrate the model. The selection of these species groups was not entirely arbitrary. Shortleaf pine is the most abundant conifer in the Central States. White oak, red oak, black oak and the hickories are the most frequently occurring species and are important commercially. Although sugar maple and black walnut occurred less frequently in my data, I included them because they are unique in their high shade tolerance and high commercial value, respectively. The descriptive statistics in Table 2 indicate the range of conditions represented in the calibration data for each species group.

CONDUCT OF THE STUDY

Coefficients for three different tree growth models were estimated for each of the species. The models were (A) the STEMS Lake States growth model, (B) a logarithmic model patterned after the Prognosis diameter growth model (Wycoff et al. 1982), and (C) a new model in which I combined features of (A) and (B).

Each model uses a different dependent variable--Model (A) predicts annual change in tree diameter, Model (B) predicts annual change in the natural log of tree diameter squared, and Model (C) predicts annual change in tree basal area. These differences presented no particular problems in model application. For this study all projection results were converted to units of annual tree basal area growth and annual tree diameter growth. Consistent with prior work with STEMS, coefficients for all models were estimated using English measurement units for all variables. Initial conditions and projection results were converted to metric units for this report.

Model (A)

The Lake States diameter growth projection model of STEMS, first introduced in the FREP Tree Growth Projection System by the USDA Forest Service (1979), has since evolved to the form described by Hahn and Leary (1979), Holdaway (unpublished), Belcher et al. (in preparation), and Holdaway and Brand (in preparation). Briefly, annual change in tree diameter is expressed as the product of that tree's estimated potential diameter growth and a modifer function that reduces potential growth to reflect the tree's situation relative to its competitors.

Potential annual tree diameter growth is expressed as the following function of initial tree diameter, crown ratio, and site index (Hahn and Leary 1979):

$$PDG = b_1 + b_2 D^{b_3} + b_4 CR \cdot SI \cdot D^{b_5} \qquad [1]$$

where

PDG = estimated potential annual diameter growth (inches at breast height)
D = tree diameter at breast height (inches)
CR = crown ratio code (a live crown of 0-10% = 1, 11-20% = 2,...71-80% = 8, 81-100% = 9)
SI = site index (feet at age 50)

Following the procedures outlined by Hahn and Leary (1979), I grouped dominant and codominant trees by 20 diameter classes, 5 crown ratio classes, and 5 site index classes (producing 500 possible categories). I combined data from categories that had only one tree with those from a similar category, and then for each nonzero category I computed potential growth as the mean growth for that category plus 1.65 standard deviations (i.e., the estimated upper 95th percentile of growth for trees in that category). Then I used nonlinear least squares to estimate coefficients for Equation [1] by using the average diameter, crown ratio code, and site index of each category as independent variables. For most species the data included few observations for trees larger than 75 cm (30 inches). Consequently, the biological constraint that as trees become very large their annual diameter growth must become very close to zero was not enforced by the model for several species. For these species I added some "artificial" observations to the potential function data. These had a site index of 99, crown ratio code of 9, a diameter equal to 80 percent of the diameter recorded in the National Register of Big Trees (American Forestry Association, 1982), and a very small potential function growth. These added observations forced the potential function to assume reasonable rates of diameter change for large diameter classes where real observations were rare or nonexistent. This had little effect on the fit of the potential function for the range of real observations.

The modifier function is a multiplier of the potential function that reduces a tree's potential growth in relation to the amount of competition that the tree encounters. The function is bounded between zero and one. The STEMS modifier function (Holdaway, unpublished) is as follows:

$$MDG = 1 - \exp(g(AD) \cdot f(R) \cdot ((BAMAX-BA)/BA)^{1/2}) \qquad [2]$$

where

MDG = proportion of potential growth actually achieved,
AD = mean stand diameter (inches),
R = relative tree diameter i.e., (tree diameter)/AD,
BA = maximum stand basal area (sq. ft.), always 250 for this study,

$$g(AD) = b_6 \ (AD + 1)^{b_7},$$

$$f(R) = b_8[1.-\exp(b_9 \cdot R)]^{b_{10}} + b_{11}$$

The dependent values (MDG) used in calibrating equation [2] are the observed diameter growth values divided by the estimated potential diameter growth from equation [1]. Because I had more observations to calibrate the modifier function than could be accommodated by the nonlinear regression program, I condensed the data into 11 basal area classes, 10 mean stand diameter classes, and 16 relative diameter classes. The mean modifier value, mean basal area, average stand diameter, and relative diameter for each class were used to calibrate equation [2]. I weighted each observation by the number of trees combined to form each class mean.

The regression coefficients for equation [2] were estimated using 2-stage nonlinear regression. First, b_6 and b_7 were estimated while holding $f(R) = 1.0$. Then, b_6 and b_7 were replaced as known constants while values of b_8, b_9, b_{10}, and b_{11} were computed. Appendix table A-1 summarizes the estimated regression coefficients for Equations [1] and [2]. Model derivation and additional details concerning calibration methodology are discussed in Holdaway (unpublished).

Model (B)

Model (B) was much easier to calibrate than model (A). No aggregation of data was required and multiple linear regression was used to estimate coefficients. Model (B) is a variation of the PROGNOSIS program diameter growth model described by Wycoff et al. (1982). That model was developed to describe the diameter growth for trees indigenous to the Northern Rocky Mountains (see Stage (1973) for additional background information).

The complete PROGNOSIS diameter growth model includes terms to account for the effects of slope, aspect, elevation, and habitat type. Because this information was not available in my Central States data, I removed those terms and replaced them with a site index term to partially compensate for the variables removed. I also eliminated the PROGNOSIS location variable, a constant used to adjust the model for a specific geographic region such as a National Forest. The following model resulted:

$$\ln(DDS) = b_1 \ln D + b_2 D^2 + b_3 CR + b_4 CR^2 + b_5 BAL/100 + b_6 CCF/100 + b_7 SI \quad [3]$$

where

ln(DDS)	= the natural log of the annual change in D^2 (in sq. ft.)
lnD	= natural log of diameter at breast height (inches)
BAL	= basal area (in sq. ft.) of trees as large or larger than the subject tree in diameter
CCF	= crown competition factor
D, CR, SI	= as previously defined

For all species except shortleaf pine 4/, I computed the crown competition factor using the equation given by Krajicek et al. (1961) for oaks and hickories. Model (B) is calibrated in logarithmic form to reduce it to a linear model and to correct for the increasing error variance associated with increasing change in diameter squared.

I used ordinary multiple linear regression to estimate the coefficients for Equation [3]. For one species, shortleaf pine, the coefficient of crown competition factor, b_6, was positive. And for three species the site index coefficient, b_7, was negative. To increase the biological credibility, I refit the model for these species with b_6 and/or b_7 set equal to zero. In no case did this procedure change the value of the original regression r^2 by more than 0.01. The estimated regression coefficients for Equation [3] are summarized in Appendix Table A-2.

Model (C)

In this model, I attempted to combine the strengths of Models (A) and (B). Like Model (A), Model (C) employs a potential and a modifier function. The potential function is a convenient way to ensure that tree growth has a reasonable upper limit. Also, use of potential and modifier functions breaks the modeling problem into two tractable components, facilitating the use of complex, nonlinear models that would be difficult to calibrate as a single, composite growth function. Model (C) also employs some of the same independent variables and variable combinations introduced by Model (B).

Model (C) takes the following form:

$$PBA = b_1 D \cdot \exp(b_2 D^2) \cdot (CR \cdot SI/1000)^{b_3} \quad [4]$$

$$MBA = 1 - \exp((1 - BA/BAMAX)^2 \cdot b_4/(BAL^{b_5} + b_6 BA)) \quad [5]$$

and

$$BAG = PBA \cdot MBA \quad [6]$$

where

PBA = annual potential basal area growth (sq. ft.) per tree
MBA = proportion of potential basal area growth actually achieved
BAG = estimated annual basal area growth per tree (sq. ft.)
D, CR, SI, BA, BAL, BAMAX are as previously defined.

4/ For shortleaf pine stands I used the equation

crown width = 2.852 + 1.529 d.b.h.

to compute crown width and crown competition factor. That equation was provided by Robert Rogers, USDA Forest Service, North Central Forest Experiment Station, Columbia, Missouri.

Two items about Model (C) are worth noting. First, the dependent variable, basal area growth per tree, is different than in either Model (A) or Model (B). A recent study by West (1980) indicates that the precision of estimates of future diameter growth is about the same when change in tree diameter is the dependent variable as when change in tree basal area is the dependent variable. But tree basal area growth is better correlated (linearly) than tree diameter growth with most of the available predictor variables. This generally makes it easier to graphically observe and to mathematically quantify the relation of observed tree basal area growth with various predictor variables than for observed diameter growth. Furthermore, volume growth is most often the quantity of greatest concern when models such as these are applied, and errors in volume growth prediction are closely tied to errors in basal area growth prediction. Using basal area growth as the dependent variable emphasizes unbiased prediction of basal area growth and should help to minimize bias in volume predictions.

Second, the relation of basal area growth to initial tree diameter in Equation [4] is similar to the relation expressed logarithmically in Model (B), Equation [3]. The function adequately described the relation of basal area growth to initial tree diameter, and it was well behaved for large diameter trees (i.e., the estimated potential basal area growth, equation [4], approached zero for large diameter trees without adding the "artificial" data necessary to control the potential diameter growth function in Model (A), equation [1]).

Potential function observations for Model (C) were selected differently than for Model (A). Model (A) used only dominant and codominant trees for potential function calibration, and as a consequence, few small diameter trees were included. Potential growth equations for Model (C) (Equation [4]) were obtained by a simpler procedure that included more small diameter trees. First, I divided all trees into 15 diameter classes and 6 crown ratio classes. Then, from each class I selected the 5 percent of the trees with the greatest basal area growth as potential observations (i.e., values of PBA). Coefficients for Equation [4] were estimated using weighted nonlinear least squares regression. The weights were $1/DBH^2$ to correct for a regression error variance proportional to diameter squared.

Data to calibrate Equation [5] were obtained by classifying the data into 30 basal area classes by 30 basal-area-in-larger-trees (BAL) classes. This reduced the number of observations to a size that could be handled by the nonlinear regression program. The mean modifier value, mean basal area, and mean BAL for each class were used as the observations. I weighted the regression by the number of cases that were combined to produce each of the class means. Coefficients for Equations [4] and [5] are given in Appendix Table A-3.

Model Comparisons

After estimating coefficients of Models (A), (B), and (C) for each of the seven species groupings, I assessed the accuracy and precision of the predictions made by each model. Because each model uses a different dependent variable and because Models (A) and (C) aggregate observations prior to regression, the goodness-of-fit statistics obtained from the regressions (e.g. r^2 and standard error of regression) do not provide a valid basis for

Table 3.--Errors in annual diameter growth and annual basal area growth per tree for calibration data.

Statistic	Species							
	Shortleaf pine	Sugar maple	Hickory	Black walnut	White oak	Red oak	Black oak	All species
Trees (No.)	1,600	325	2,464	264	3,743	1,041	3,554	12,991
Annual DBH Growth (cm x 100)								
Observed mean	33	27	19	27	30	42	38	31
Mean Residual 1/								
Model (A)	1	-2	0	-1	-1	-3	-2	-1
Model (B)	-5	-5	-2	-9	-2	-1	0	-2
Model (C)	5	2	2	4	3	4	4	3
Root MSR 2/								
Model (A)	20	17	13	25	15	19	17	16
Model (B)	20	22	15	26	16	20	17	17
Model (C)	20	18	13	25	15	19	17	16
Annual Basal Area Growth (cm^2)								
Observed mean	11	11	7	11	13	21	17	13
Mean Residual 1/								
Model (A)	-1	-1	-2	-2	-2	-3	-2	-2
Model (B)	-2	-3	-1	-5	-2	-2	-2	-2
Model (C)	0	-1	0	0	0	0	0	0
Root MSR 2/								
Model (A)	7	9	6	12	8	12	9	8
Model (B)	7	11	6	14	9	12	9	9
Model (C)	7	9	5	12	8	11	9	8

1/ Mean residual = Σ(Predicted growth - actual growth)/n

2/ Root mean squared residual = $(\Sigma(\text{Predicted growth - actual growth})^2/n)^{1/2}$.

comparing the different models. Instead, I used each model to predict annual diameter growth and annual basal area growth for each tree in the calibration data. Then I summarized the mean residual (predicted minus actual growth) and the root mean squared residual 5/ of annual diameter growth and basal area growth predictions for the calibration data by species and by model (Table 3).

5/ The root mean squared residual =

$$(\Sigma(\text{predicted - actual growth})^2/(\text{total number of trees}))^{1/2}$$

Table 4.--Errors in annual diameter growth and annual basal area growth per tree for validation data.

Statistic	Species							
	Shortleaf pine	Sugar maple	Hickory	Black walnut	White oak	Red oak	Black oak	All species
Trees (No.)	613	110	874	118	1,143	256	1,075	4,189
Annual DBH Growth (cm x 100)								
Observed mean	34	30	19	26	31	42	39	31
Mean Residual 1/								
Model (A)	1	-7	0	2	-2	-2	-4	-1
Model (B)	-5	-10	-2	-7	2	-2	-1	-2
Model (C)	4	-2	2	7	2	5	5	3
Root MSR 2/								
Model (A)	21	19	13	23	15	20	17	17
Model (B)	20	26	14	24	16	21	17	17
Model (C)	20	19	13	22	15	20	17	17
Annual Basal Area Growth (cm^2)								
Observed mean	12	14	7	10	14	22	18	14
Mean Residual 1/								
Model (A)	-1	-5	-1	0	-2	-3	-3	-2
Model (B)	-3	-6	-1	-3	-2	-3	-2	-2
Model (C)	0	-4	0	1	0	-1	-1	0
Root MSR 2/								
Model (A)	8	11	5	9	8	14	9	8
Model (B)	8	12	6	9	8	14	9	8
Model (C)	7	10	5	8	8	13	9	8

1/ Mean residual = Σ(Predicted growth - actual growth)/n

2/ Root mean squared residual = $(\Sigma(\text{Predicted growth - actual growth})^2/n)^{1/2}$.

I chose to present these results in terms of error in basal area and diameter growth because these two dependent variables are typically most important when models are applied. The mean growth residual (predicted minus actual growth) indicates the bias of each model. The square root of the mean square residual is a combined estimate of the bias and precision of each model's estimates. To get a more complete picture of each model's capabilities, I also computed the same statistics for each model using the validation data--the one-fourth of the plots not used to calibrate the model (Table 4).

RESULTS AND DISCUSSION

The outstanding feature of the results is the similarity in the projection accuracy and precision for all three models (Tables 3 and 4). Although the models were formulated in different ways, they produce similar estimates of annual growth for the range of species and conditions considered.

Some differences were found in model performance. For example, Models (A) and (B) underestimate diameter growth for most species both in the calibration and validation data. Model (C) overestimates diameter growth to a greater degree than Models (A) and (B) underestimate it. But the models differ little in their root mean square residuals for diameter growth (Table 3 and Table 4). The mean residual basal area growth for all species was generally much smaller for Model (C) than for Models (A) or (B). This was expected, because basal area growth was the dependent variable only in Model (C). As with diameter growth, the root mean squared residuals for basal area growth for all species were similar for all three models. But the magnitude of the differences among models was small and in many applications these differences would hold little practical significance.

Another notable feature is the magnitude of the root mean square residuals (Tables 3 and 4). Root mean square residual values for both diameter and basal area growth generally range from 50 to 60 percent of the mean growth. This indicates that a significant share of the variability in diameter and basal area growth remains unaccounted for regardless of the model used. This unexplained variation can be placed in the context of an r^2 for diameter growth and basal area growth, using the fact that $r^2 = 1 -$ (error sum of squares/total sum of squares). For the calibration data, the r^2 estimate for diameter growth for all three models ranged from zero to 0.13. For calibration data basal area growth, the estimated r^2 ranged from 0.01 to 0.51. Estimated r^2 values for diameter growth were highest for Model (A). The highest basal area growth r^2 estimates come from Model (C).

CONCLUSIONS

For the range of conditions considered, the three models discussed in this paper predict annual diameter and basal area growth with essentially equal accuracy and precision. Model performance differs slightly by species and by choice of dependent variable.

It is instructive to speculate why such similarities were found in the numerical performance of the models considered in this study and why a large share of the variability in tree growth was not accounted for by any of the models. One possibility is that all three models are poorly formulated, omit important biological considerations, and/or have other defects that render them inappropriate and ineffective for prediction. A more plausible alternative is that these three models are members of a large group of similarly formulated models that would all produce similar growth estimates for the range of conditions and time span considered. Alternative models that would reduce the residual variations in diameter and/or basal area growth for these data could almost certainly be found. But gains in accuracy and precision would likely be modest without first increasing the accuracy with which the independent variables are measured and/or incorporating new independent variables not included in Models (A), (B), or (C).

None of the three models analyzed in this paper is universally superior to the others in forecasting annual individual tree diameter and basal area growth for the range of species, conditions, and approximately 11-year time span considered. Therefore, factors other than precision or accuracy of predictions should also be considered in selecting which one of these models to use. These other factors include cost of calibration, ease of calibration, compatibility with available data, ability of the model to provide reasonable estimates under uncommon or unusual conditions, ability of the model to provide reasonable estimates for long projection periods, units in which output information will be reported, and compatibility with existing application programs.

If diameter or basal area growth will be estimated for a period of only about 10 years, there appears to be little analytical basis to prefer one model over the others. Although Model (A) had the smallest mean residual diameter growth and Model (C) had the smallest mean residual basal area growth for short projection periods, the slight gains from using Model (A) or (C) might well be offset by the speed and simplicity of calibrating a linear model such as Model (B). When projecting growth over longer time spans (more than 10 years), even small errors and biases will be compounded and may become important. Thus, the importance of using the least biased and most precise growth estimation model is greatly increased. Such situations favor the application of explicitly constrained models such as Models (A) and (C). Model (A) is preferred for when estimates of future tree diameter are primarily important, and Model (C) is preferred when predictions of tree basal area are most important.

ACKNOWLEDGEMENTS

I am very grateful to the following people for their cooperation in supplying the data essential to this study: A. Jerry Ostrom, Robert Rogers, Ivan Sander, Richard Schlesinger, and W. Brad Smith of the North Central Forest Experiment Station; John W. Moser, Jr., W. L. Mills, and W. Lloyd Fix of Purdue University; Robert Gregory and Robert Brooks of the Tennessee Valley Authority; the staff of the Pioneer Forest; and George Gertner and Dieter Pelz of the University of Illinois.

LITERATURE CITED

American Forestry Association. 1982. National Register of Big Trees. American Forests 88(4):19-47.

Belcher, D. M., M. R. Holdaway, and G. J. Brand. In preparation. STEMS--the Stand and Tree Evaluation and Modeling System. U. S. Forest Service General Technical Report NC-70. North Central Forest Experiment Station, St. Paul, Minnesota.

Dale, M. B. 1975. Individual tree growth and simulation of stand development of an 80-year old white oak stand. p. 49-63 In A. R. Ek, J. W. Balsinger, and L. C. Promnitz. Forest Modeling and Inventory: Selected papers from the 1973 and 1974 meetings of the midwest Mensurationists. School of Natural Resources, College of Agricultural and Life Sciences, University of Wisconsin, Madison, Wisconsin.

Hahn, J. T., and R. A. Leary. 1979. Potential diameter growth functions. In A generalized forest growth projection system applied to the Lake States region. U. S. Department of Agriculture Forest Service, General Technical Report NC-49, p. 22-26. U. S. Department of Agriculture Forest Service, North Central Forest Experiment Station, St. Paul, Minnesota.

Holdaway, M. R. 1981. Modeling the effect of social position on tree diameter growth; a new modifier function for the FREP tree growth projection system. 21 p. Unpublished study report, available from author, North Central Forest Experiment Station, St. Paul, Minnesota 55108.

Holdaway, M. R., and G. J. Brand. In preparation. An evaluation of the STEMS tree growth projection system. U. S. Department of Agriculture Forest Service, North Central Forest Experiment Station, St. Paul, Minnesota.

Jakes, P. J., and W. B. Smith. 1980. Predicted yields from selected cutting prescriptions in northern Minnesota. U. S. Department of Agriculture Forest Service Research Paper NC-188. 29 p. U. S. Department of Agriculture Forest Service, North Central Forest Experiment Station, St. Paul, Minnesota.

Krajicek, J. E., K. A. Brinkman, and S. F. Brinkman. 1961. Crown competition--a measure of density. Forest Science 7:35-42.

Munro, D. D. 1974. Forest growth models--a prognosis. In Growth models for tree and stand simulation. J. Fries (editor). Royal College of Forestry Research Notes Number 70, p. 7-21. Stockholm.

Raile, G. K., and W. B. Smith. 1982. A simulated inventory update: Wisconsin's 1979 timber resource. U. S. Department of Agriculture Forest Service Research Paper NC-209. 40 p. U. S. Department of Agriculture Forest Service, North Central Forest Experiment Station, St. Paul, Minnesota.

Stage, A. R. 1973. Prognosis model for stand development. U. S. Department of Agriculture Forest Service Research Paper INT-137. 32 p. U. S. Department of Agriculture Forest Service Intermountain Forest and Range Experiment Station, Ogden, Utah.

U. S. Department of Agriculture, Forest Service. 1979. A generalized forest growth projection system applied to the Lake States region. U. S. Department of Agriculture Forest Service, General Technical Report NC-49. 96 p. North Central Forest Experiment Station, St. Paul, Minnesota.

West, P. W. 1980. Use of diameter increment and basal area increment in tree growth studies. Canadian Journal of Forest Research 10: 71-77.

Wycoff, W. R., N. L. Crookston, and A. R. Stage. 1982. User's guide to the STAND Prognosis Model. U. S. Department of Agriculture Forest Service Research Paper INT-133. U. S. Department of Agriculture Forest Service Intermountain Forest and Range Experiment Station, Ogden, Utah.

APPENDIX

Table A1.--Estimated regression coefficients for Model (A).

Species	No. of classes	Potential Function Regression Coefficients1/					
		1 b_1	b_2	b_3	b_4	b_5	r^2
Shortleaf Pine	132	.30000000E+00	-.20389169E-01	.10000000E+01	.53670726E-03	.10000000E+00	.27
Sugar Maple	64	.15862115E+00	-.66669172E-04	.23163591E+01	.12567438E-03	.40249054E+00	.15
Hickory	221	.22417563E+00	-.68340434E-02	.10000000E+01	.76063313E-04	.10000000E+00	.11
Black Walnut	64	.12997765E+00	-.86406939E-02	.12113030E+01	.94399096E-03	.10000000E+00	.43
White Oak	284	.17930589E+00	-.17063805E-02	.12859333E+01	.26715387E-03	.10000000E+00	.32
Red Oak	192	.21671760E+00	-.88323315E-04	.20269259E+01	.29372392E-03	.10000000E+00	.30
Black Oak	317	.21800551E+00	-.51252219E-03	.16995493E+01	.27133217E-03	.10000000E+00	.32

Species	No. of classes	Modifier Function Regression Coefficients2/						
		b_6	b_7	b_8	b_9	b_{10}	b_{11}	r^2
Shortleaf Pine	216	1.5075755E+00	-7.5181638E-01	1.4599902E+00	-1.4452141E+00	2.1309914E+00	0E+00	.24
Sugar Maple	156	3.0519868E-01	4.9593266E-02	1.0980438E+00	-1.9736924E+00	3.3060159E+00	2.2928548E-01	.15
Hickory	308	2.9803037E-01	-5.5042402E-02	2.4102227E+00	-3.4379638E+00	9.0054714E-01	0E+00	.38
Black Walnut	133	3.2484602E-01	-1.2591706E-01	1.0086176E+00	-2.8064966E+00	1.7011626E+01	3.5242397E-01	.00
White Oak	379	4.6929616E-01	-1.2285000E-01	1.3293996E+00	-1.5942786E+00	2.3511348E+00	0E+00	.41
Red Oak	258	6.3358742E-01	-2.2915142E-01	1.2468124E+00	-1.8858640E+00	3.0215077E+00	0E+00	.00
Black Oak	372	4.8323332E-01	-9.9469517E-03	1.2765865E+00	-1.7446142E+00	2.5931441E+00	0E+00	.25

1/ For the model

$$PDG = b_1 + b_2 D^{b_3} + b_4 CR \cdot SI \cdot D^{b_5}$$

where
PDG = potential annual tree diameter growth (in inches)
D = tree diameter at breast height (in inches)
CR = crown ratio code (a live crown of 0-10% = 1, 11-20% = 2,..., 71-80% = 8, 81-100% = 9)
SI = site index (in feet) at age 50
subject to the constraints that $b_1 \leq 0.03$ and $b_5 \geq 0.1$.

2/ For the model

$$MDG = 1 - \exp(g(AD) \cdot f(R) \cdot ((250-BA)/BA)^{1/2})$$

where
MDG = modifier value for potential diameter growth function

$$g(AD) = b_6 (AD+1)^{b_7}$$

$$f(R) = b_8 [1.-\exp(b_9 \cdot R)]^{b_{10}} + b_{11}$$

AD = average stand diameter (in inches)
R = tree diameter divided by the average stand diameter
BA = stand basal area
Subject to the constraint $b_{11} \geq 0.0$

Table A2.--Estimated regression coefficients for Model (B)

Species	No. of trees	Regression Coefficients 1/ b_1	b_2	b_3	b_4
Shortleaf Pine	1,604	.15435945E+00	.99344803E-03	.37098150E+00	-.25142259E-01
Sugar Maple	330	.13721430E+01	-.31725869E-02	-.88125029E+00	.88504414E-01
Hickory	2,493	.11815767E+01	-.14508589E-02	-.61986286E+00	.69806088E-01
Black Walnut	264	.30776760E+00	.69539842E-03	.98456205E-01	.69806088E-02
White Oak	3,763	.99513009E+00	-.15152433E-02	.26462334E+00	.30119088E-01
Red Oak	1,042	.12019160E+01	-.80342429E-03	-.48385041E+00	.44513842E-01
Black Oak	3,562	.99000794E+00	-.85980682E-03	-.312144664+01	.36377516E-01

Species	Regression Coefficients b_5	b_6	b_7	r^2
Shortleaf Pine	-.26866783E+01	0	0	.20
Sugar Maple	-.17958018E+01	-.28741398E+00	.14105244E-01	.29
Hickory	-.20662162E+01	-.20716630E+00	0	.38
Black Walnut	-.14269341E+01	-.24959532E+00	0	.08
White Oak	-.17731819E+01	-.11852520E+00	0	.46
Red Oak	-.85322089E-00	-.18157731E+00	.41184281E-02	.66
Black Oak	-.10417776E+01	-.10347077E+00	.48184488E-04	.74

1/ For the model

$$\ln(D^2) = b_1 D + b_2 \ln D + b_3 CR + b_4 CR^2 + b_5\, BAL/100 + b_6\, CCF/1000 + b_7 SI$$

where

$\ln(D^2)$ = natural log of annual change in diameter squared (in square feet)
D = diameter at breast height (inches)
lnD = natural log of diameter
CR = crown ratio code (a live crown of 0-10% = 1, 11-20% = 2,...71-80% = 8, 81-100% = 9)
BAL = basal area of trees with DBH equal to or larger than the subject tree (sq.ft. acre)

Table A3.--Estimated regression coefficients for Model (C)

Species	No. of classes	Potential Function Regression Coefficients 1/			
		b_1	b_2	b_3	r^2
Shortleaf Pine	189	.68836083E-02	-.58490492E-02	.13053398E+00	.19
Sugar Maple	72	.36678837E-02	-.75026280E-03	.23092751E+00	.64
Hickory	162	.47895279E-02	-.73746470E-03	.28394553E+00	.62
Black Walnut	59	.49421043E-02	-.23799570E-02	.18493976E+00	.28
White Oak	233	.49407820E-02	-.49688670E-03	.17782107E+00	.80
Red Oak	149	.69290622E-02	-.31272194E-03	.30480406E+00	.78
Black Oak	224	.41661151E-02	-.65470419E-03	0E+00	.79

Species	No. of classes	Modifier Function Regression Coefficients 2/			
		b_4	b_5	b_6	r^2
Shortleaf Pine	216	.79938903E+01	.70928467E+00	0E+00	.48
Sugar Maple	156	.20492911E+01	.24907300E+00	0E+00	.09
Hickory	308	.11593701E+01	.30415823E+00	0E+00	.24
Black Walnut	133	.89004047E+00	.63435677E-01	0E+00	.01
White Oak	379	.14965407E+01	.24374211E+00	.50900329E-02	.44
Red Oak	258	.90690637E+00	.85490101E-01	0E+00	.04
Black Oak	372	.17186851E+01	.22930419E+00	.14806753E-02	.42

1/ For the model

$$PBA = b_1 D \cdot \exp(b_2 \cdot D^2) \cdot (CR \cdot SI/1000)^{b_3}$$

where
PBA = potential tree basal area growth
D = diameter at breast height (inches)
CR = crown ratio class (0-10% = 1, 11-20% = 2,...71-80% = 8, 100% = 9)
SI = site index (in feet) at age 50.
Subject to the constraint $b_3 \geq 0.0$

2/ For the model

$$MBA = 1 - \exp[(-b_4/(BAL^{b_5} + b_6 \cdot BA))(1-BA/250)^{1/2}]$$

where
MBA = modifier value for Potential basal area growth function
BAL = basal area of trees as large or larger than the subject tree (square feet/acre)
BA = total stand basal area (square feet/acre)
Subject to the constraint $b_6 \geq 0.0$